

1. Solve the initial value differential equation.

$$\frac{dy}{dx} - 2xy^2 = 0 \quad y(0) = 3.$$

answer: $\frac{1}{y^2} dy = 2x dx$ So $y(x) = -\frac{1}{x^2 - 1/3} = -\frac{3}{3x^2 - 1}$

2. Solve the initial value differential equation.

$$y' = (2x + 3)\sqrt{y} \quad y(0) = 2.$$

answer: $\frac{1}{\sqrt{y}} dy = (2x + 3) dx$ So $y(x) = \frac{1}{4}(x^2 + 3x + \sqrt{8})^2$

3. Show that $P(t) = \frac{1}{1 + e^{-t}}$ solves the logistics equation $\frac{dP}{dt} = P(1 - P)$

answer: $\frac{dP}{dt} = P'(t) = -\frac{e^{-t}}{(1 + e^{-t})^2}$ And

$$P(1 - P) = \left(\frac{1}{1 + e^{-t}}\right) \left(1 - \frac{1}{1 + e^{-t}}\right) = \left(\frac{1}{1 + e^{-t}}\right) \left(-\frac{e^{-t}}{1 + e^{-t}}\right) = -\frac{e^{-t}}{(1 + e^{-t})^2}$$

4. Determine the solution to the logistics problem: $\frac{dP}{dt} = 0.1P(1000 - P)$ where $P(0) = 100$.

answer: $P(t) = \frac{1000}{1 + 9e^{-100t}}$

5. When a murder is committed, the body, originally at 37°C , cools according to the differential equation $\frac{dH}{dt} = -k(H - 20)$, where k is positive. Suppose that after two hours the temperature is 35°C .

- (a) Solve the differential equation for $H(t)$, the temperature at time t .

answer given $H(0) = 37$: $H(t) = 20 + 17e^{-kt}$

final answer given $H(2) = 35$: $H(t) = 20 + 17e^{0.5(\ln(15/17))t} = 20 + 17e^{-0.06258t}$

(b) What happens to the temperature of the body in the long run?

answer: It cools to the temperature $T = \lim_{t \rightarrow \infty} H(t) = 20^\circ$.

(c) If the body is found at 4 p.m. at a temperature of 30°C , when was the murder committed?

answer: $\ln(10/17) = 0.5t \ln(15/17) \Rightarrow t \approx 8.5$ hr. Therefore the murder was committed around 7 : 30 a.m.

6. Suppose the water reservoir holds 100 million gallons of water and supplies a city with 1 million gallons a day. The reservoir is partly refilled by a spring which provides 0.9 million gallons a day and the rest of the water, 0.1 million gallons a day, comes from a run-off from the surrounding land. The spring is clean, but the run-off contains salt with a concentration of 0.0001 pounds per gallon. Assume that there was no salt in the reservoir initially and that the reservoir is well mixed. Find the amount of salt in the reservoir as a function of time.

Answer: Let $y(t)$ = amount of salt in reservoir at time t .

Rate In = $900,000 \text{ gl/dy} \times 0 \text{ lb/gl} + 100,000 \text{ gl/dy} \times 0.0001 \text{ lb/gl} = 10 \text{ lb/day}$

Rate Out = $1,000,000 \text{ gl/dy} \times \frac{y}{100,000,000} \text{ lb/gl} = \frac{y}{1000} \text{ lb/day}$

$\frac{dy}{dt} = 10 - \frac{y}{1000}$ Therefore: $\frac{dy}{dt} = -\frac{1}{1000}(y - 10000)$ And:

$y(t) = 10000 - 10000e^{-\frac{1}{1000}t}$

7. Suppose your bank offers 5% continual annual interest on the balance of your certificate of deposit but you make a continual withdrawal of \$400 per year.

(a) Write a differential equation involving the balance of your CD at any time t .

Let $B(t)$ = balance at time t .

Answer: $\frac{dB}{dt} = 0.05B - 400 = \frac{1}{20}(B - 8000)$

(b) Solve this differential equation and give the minimum initial deposit so that your funds are not depleted over time.

Answer: $B(t) = 8000 + Ae^{-0.05t}$

Minimal Initial Deposit is \$8000.00

(c) Determine $B(t)$ if you initially deposit \$12,000 and do not deposit again.

Answer: $B(t) = 8000 + 4000e^{0.05t}$