

1. Determine the volume of the solid formed by rotating the region bounded by $y = 2^x$ and $y = x^2$ for $0 \leq x \leq 2$ about the x -axis.

$$\pi \int_0^2 (4^x - x^4) dx = \pi \left(\frac{15}{\ln 4} - \frac{32}{5} \right)$$

2. Determine the volume of the solid formed by rotating the region bounded by the x -axis and the curve $y = 2e^{-x}$ for $0 \leq x \leq 2$ about the line $y = -1$.

$$\pi \int_0^2 [(2e^{-x} + 1)^2 - 1] dx = \pi (-2e^{-4} - 4e^{-2} + 4)$$

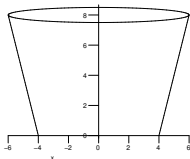
3. Determine the volume of the solid formed by rotating the region bounded by the y -axis, the x -axis, and the curve $y = \cos x$ for $0 \leq x \leq \frac{\pi}{2}$ about the line $x = -1$.

$$2\pi \int_0^{\pi/2} (x + 1) \cos x dx = \pi^2$$

4. Determine the volume of the solid formed by rotating the region bounded by the y -axis, the line $y = x$, and the curve $y = 1 + \sqrt{1 - x^2}$ for $0 \leq x \leq 1$ about the y -axis.

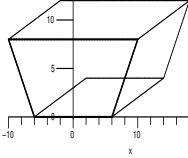
$$2\pi \int_0^1 (x + x\sqrt{1 - x^2} - x^2) dx = \pi$$

5. Set up the integral (do not evaluate) to determine the work to empty a full tank of water that is the shape of a right circular cone having bottom radius 4 ft and top radius 6 ft and height 8 ft. Water has density approximately 62.4 lb/ft^3 .



$$W = 62.5\pi \int_0^8 \left(\frac{y + 16}{4} \right)^2 (8 - y) dy.$$

6. Set up the integral (do not evaluate) to determine the work to empty a full swimming pool of water that has a rectangular bottom with length 12 ft and width 10 ft and rectangular top with length 20 ft and width 10 ft. The pool is 8 ft underground. Water has density approximately 62.4 lb/ft^3 .



$$W = 62.4 \int_0^8 10(y + 12)(8 - y) dy.$$

7. Set up the integral (do not evaluate) to determine the force on the trapezoidal side of the pool that is shown above when the pool is full of water.

$$F = 62.4 \int_0^8 (y + 12)(8 - y) dy.$$

8. Set up the integral to determine the force on the side of a fish tank that has a length of 4 ft and a height of 2 ft if the tank is full.

$$F = 62.4 \int_0^2 4(2 - y) dy.$$

9. Set up the integral to determine the arclength of the curve $y = x^2 + 3x + 1$ on $0 \leq x \leq 2$ (do not evaluate).

$$AL = \int_0^2 \sqrt{10 + 12x + 4x^2} dx.$$

10. Set up the integral to determine the arclength of the curve $y = \sin(2x)$ on $0 \leq x \leq \frac{\pi}{2}$ (do not evaluate).

$$AL = \int_0^{\pi/2} \sqrt{1 + 4 \cos^2 x} dx.$$

11. Determine the average value of the function $f(x) = \frac{3x}{1 + x^2}$ on $0 \leq x \leq 5$.

$$AV = \frac{1}{5} \int_0^5 \frac{3x}{1 + x^2} dx = \frac{3}{10} \ln 26.$$

12. Determine the average value of the function $f(x) = \frac{8}{1 + 4x^2}$ on $0 \leq x \leq \frac{1}{2}$.

$$AV = 2 \int_0^{1/2} \frac{8}{1 + 4x^2} dx = 8 \arctan(1) = 2\pi.$$

13. Evaluate the integral if it converges. Show divergence otherwise.

$$(a) \int_0^{\infty} 3xe^{-4x^2} dx = -\frac{3}{8}e^{-4x^2} \Big|_{x=0}^{x=t \rightarrow \infty} = \frac{3}{8}. \quad \text{Converges.}$$

$$(b) \int_0^{\infty} \frac{4}{1+9x^2} dx = \frac{4}{3} \arctan(3x) \Big|_{x=0}^{x=t \rightarrow \infty} = \frac{2\pi}{3}. \quad \text{Converges.}$$

$$(c) \int_1^4 \frac{4}{(x-1)^2} dx = -\frac{4}{x-1} \Big|_{x=t \rightarrow 1}^{x=4} = \infty. \quad \text{Diverges.}$$

$$(d) \int_1^4 \frac{x}{x^2-1} dx = \frac{1}{2} \ln(x^2-1) \Big|_{x=t \rightarrow 1}^{x=4} = \infty. \quad \text{Diverges.}$$

14. Solve the initial value differential equation for $y(t)$:

$$\frac{dy}{dt} = ty^3 \quad y(0) = 1$$

$$y(t) = \frac{1}{\sqrt{1-t^2}}$$

15. Solve the initial value differential equation for $y(t)$:

$$\frac{dy}{dt} = t\sqrt{y-1} \quad y(0) = 5$$

$$y(t) = 1 + \left(\frac{1}{4}t^2 + 2\right)^2.$$

16. Use Euler's Method to approximate $y(1)$ if $\frac{dy}{dx} = y + x$ with $y(0) = \frac{1}{2}$ and $\Delta x = \frac{1}{2}$.

x_n	y_n	dy/dx	$y_{n+1} = y_n + dy/dx(\Delta x)$
0	1/2	1/2	1/2 + 1/2(1/2)
1/2	3/4	5/4	3/4 + 5/4(1/2)
1	11/8		

17. Use Euler's Method to approximate $y(1)$ if $\frac{dy}{dx} = \frac{1}{2}y - x$ with $y(0) = 1$ and $\Delta x = \frac{1}{2}$.

x_n	y_n	dy/dx	$y_{n+1} = y_n + dy/dx(\Delta x)$
0	1	1/2	1 + 1/2(1/2)
1/2	5/4	1/8	5/4 + 1/8(1/2)
1	21/16		

18. A tank contains 1000 L of pure water. Brine containing 0.05 kg of salt per liter of water enters the tank at a rate of 5 liters per minute. Brine that contains 0.04 kg of salt per liter of water enters the tank at a rate of 10 liters per minute. The solution is kept thoroughly mixed and drains at a rate of 15 liters per minute.

(a) Determine a differential equation involving the amount of salt in the tank at a given time, t .

$$\frac{dy}{dt} = .05(5) + .04(10) - \frac{y}{1000}(15) = \frac{65}{100} - \frac{15y}{1000} = -\frac{15}{1000} \left(y - \frac{650}{15} \right)$$

(b) Solve the differential equation giving the amount of salt in the tank at any time, t .

$$y(t) = \frac{650}{15} - \frac{650}{15} e^{(-15/1000)t}$$

19. A tank contains 800 L of pure water. Brine containing 0.01 kg of salt per liter of water enters the tank at a rate of 5 liters per minute. Brine that contains 0.02 kg of salt per liter of water enters the tank at a rate of 10 liters per minute. The solution is kept thoroughly mixed and drains at a rate of 15 liters per minute.

(a) Determine a differential equation involving the amount of salt in the tank at a given time, t .

$$\frac{dy}{dt} = .01(5) + .02(10) - \frac{y}{800}(15) = \frac{1}{4} - \frac{15y}{800} = -\frac{15}{800} \left(y - \frac{200}{15} \right)$$

(b) Solve the differential equation giving the amount of salt in the tank at any time, t .

$$y(t) = \frac{200}{15} - \frac{200}{15} e^{(-15/800)t}$$

20. Solve the initial value first order linear differential equation:

$$y' = 2y + x \qquad y(0) = 1.$$

$$y = -\frac{1}{2}x - \frac{1}{4} + \frac{3}{4}e^{-2x}$$

21. Give the general solution to the second order differential equation.

$$y'' + 2y' + 5y = 0$$

$$y = e^{-x} (M \cos(2x) + N \sin(2x))$$

22. Give the solution to the initial value second order nonhomogeneous differential equation:

$$y'' + 2y' + y = 4 \sin x \qquad y(0) = 0 \qquad y'(0) = 0.$$

$$y = 2e^{-x} + 2xe^{-x} - 2 \cos x$$

23. Tell whether the series converges or diverges and justify your answer by showing reason by a valid test.

(a) $\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$

Converges by comparison test (to improper integral) $\int_1^{\infty} \frac{1}{x^2 + 1} dx = \frac{\pi}{4}$

(b) $\sum_{n=0}^{\infty} \frac{n^2}{2^n}$

Converges by the ratio test: $\lim_{n \rightarrow \infty} \left| \frac{(n+1)^2}{2^{n+1}} \cdot \frac{2^n}{n^2} \right| = \frac{1}{2} < 1$

24. Determine the given sum:

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{n!} = \underline{\underline{e^{-3}}}$

(b) $9 - 3 + 1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \dots = \underline{\underline{\frac{27}{4}}}$

25. Determine the interval and radius of convergence of the given series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n 2^n}$$

$$I : -2 < x \leq 2 \qquad R = 2$$

26. Determine the Taylor Series about $x = 0$ for:

(a) $f(x) = x^2 e^{-x}$

$$T(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+2}}{n!}$$

$$(b) g(x) = \frac{1}{1 + 9x^2}$$

$$T(x) = \sum_{n=0}^{\infty} (-9)^n x^{2n}$$

$$(c) h(x) = \frac{1}{3} \arctan(3x)$$

$$T(x) = \sum_{n=0}^{\infty} \frac{(-9)^n x^{2n+1}}{2n+1}$$

27. Use the binomial expansion to determine the first 3 terms of the MacLaurin series representing the function:

$$f(x) = \frac{1}{\sqrt[3]{1+x}}$$

$$T(x) = \sum_{n=0}^{\infty} \binom{-1/3}{n} x^n \approx 1 - \frac{1}{3}x + \frac{2}{9}x^2$$

28. Write out the first three NONZERO terms to the MacLaurin series of

$$f(x) = e^x \cos x$$

(Determine the c 's to $e^x \cos x = c_0 + c_1x + c_2x^2 + c_3x^3 + \dots$)

$$T_3(x) = 1 + 1x + 0x^2 - \frac{1}{3}x^3$$

29. Use series to approximate $\int_0^1 \frac{e^x - 1}{x} dx$. (Write out 4 terms but do not add them.)

$$S_4 = 1 + \frac{1}{4} + \frac{1}{24} + \frac{1}{96}$$

30. If $\vec{a} = \langle -3, 2, 5 \rangle$ and $\vec{b} = \langle 6, 2, 3 \rangle$, calculate:

$$(a) \vec{a} + \vec{b}, |\vec{a}|, \vec{a} \cdot \vec{b}, \vec{a} \times \vec{b}, \text{ and } \cos \theta = \text{angle between } \vec{a} \text{ and } \vec{b}.$$

$$\vec{a} + \vec{b} = \langle 3, 4, 8 \rangle \quad |\vec{a}| = \sqrt{38} \quad \vec{a} \cdot \vec{b} = 1 \quad \cos \theta = \frac{1}{7\sqrt{38}}$$

$$\vec{a} \times \vec{b} = \langle -4, 39, -18 \rangle$$

- (b) Π , the equation of the plane parallel to both \vec{a} and \vec{b} containing the point $P(-1, 9, 2)$.

$$-4(x+1) + 39(y-9) - 18(z-2) = 0$$

(c) \vec{P} , the projection vector of \vec{a} onto \vec{b} .

$$\vec{P} = \frac{1}{49} \langle 6, 2, 3 \rangle = \left\langle \frac{6}{49}, \frac{2}{49}, \frac{3}{49} \right\rangle$$

31. Change from rectangular coordinates to cylindrical coordinates:

$$\begin{array}{llll} P(1, \sqrt{3}, 2) & Q(4, -4, 8) & z^2 + 4x^2 + 4y^2 = 9 & z = x^2 - y^2 \\ P(2, \pi/3, 2) & Q(4\sqrt{2}, 7\pi/4, 8) & z^2 + 4r^2 = 9 & z = r^2 \cos 2\theta \end{array}$$

32. Change from rectangular coordinates to spherical coordinates:

$$\begin{array}{llll} P(1, \sqrt{3}, 2\sqrt{3}) & Q(7, 7, 0) & z = x^2 + y^2 & 4x^2 + 4y^2 + 4z^2 + 2z = 5 \\ P(4, \pi/3, \pi/6) & Q(7\sqrt{2}, \pi/4, \pi/2) & \rho = \cot \phi \csc \phi & 4\rho^2 + 2\rho \cos \phi = 5 \end{array}$$

33. Change from spherical coordinates to rectangular coordinates:

$$\begin{array}{llll} P(1, 0, \pi/4) & Q(3, \pi/3, 5\pi/6) & \rho = 2 \cos \phi & \phi = \pi/6 \\ P(\sqrt{2}/2, 0, \sqrt{2}/2) & Q(3/4, 3\sqrt{3}/4, -3\sqrt{3}/2) & & \\ x^2 + y^2 + (z - 1)^2 = 1 & z = \sqrt{3x^2 + 3y^2} & & \end{array}$$

34. Sketch the graph of the given surface and identify it.

(a) $f(x, y) = 6 - 2x - 3y$ PLANE

(b) $f(x, y) = 4x^2 + y^2$ ELLIPTIC PARABALOID

(c) $x^2 - 2x + y^2 + z^2 + 4z + 1 = 0$ SPHERE

(d) $f(x, y) = \sqrt{16 - x^2 - 4y^2}$ HEMISPHERE

(e) $x^2 + 3y^2 = 9$ ELLIPTIC CYLINDER