1. Find the following

   (a) $\int xe^x \, dx$

   (b) $\int_0^{\frac{\pi}{3}} \sin x\sqrt{1 + 2\cos x} \, dx$

   (c) Find $\int_4^8 \frac{3}{\sqrt{x-4}} \, dx$

   (d) Find the average value of $f(x) = \frac{1}{x^2}$ on the interval $[1, 2]$.

   (e) The function f is defined be $f(x) = \int_0^{x^5} e^{-t^2} \, dt$. Find $f'(x)$.

2. The area bounded by the x-axis and the curve $y = \sqrt{x-1}$, $1 \leq x \leq 5$ is revolved about the y-axis.

   (a) Set up but do not evaluate, an integral for the volume of revolution generated using cylindrical shells.

   (b) Set up but do not evaluate, an integral for the volume of revolution generated using annular rings or washers.

3. Use Euler's method with step size h=0.1 to compute the approximate y-values y_1, y_2, y_3 and y_4 of the solution of the initial-value problem $y' = x + y$, $y(0) = 1$. Show your work.

n	x_n	y_n	
0	0	1	
1			
2			
3			
4			

4. A water tank in the shape of an inverted right circular cone has a radius of 4 feet and a height of 8. It is filled to a height of 6 feet. Set up, but do not evaluate, an integral for the work done in pumping the water to the top of the tank.

5. Find the solution to the initial-value problem $y' = 2xy - y$, $y(0) = 2$.

6. A model for learning is $\dfrac{dP}{dt} = k(M - P)$ where P is the performance, M is the maximum level of performance, and k is a positive constant. Solve this equation for $P(t)$ and find the $\lim_{t\to\infty} P(t)$.

7. (a) Determine whether $\sum_{n=1}^{\infty} \dfrac{n^2}{n^3 - 2}$ converges or diverges.

   (b) Find the sum of the series $\sum_{n=0}^{\infty} \dfrac{3^{2n+2}}{2^{4n+1}}$.

   (c) Find the sum of the series $\sum_{n=0}^{\infty} \dfrac{3^n}{n!}$.

8. Find the radius of convergence for the power series $\sum_{n=1}^{\infty} \dfrac{(2x+1)^n}{n}$.

9. (a) Write $f(x) = \frac{1}{1+x^2}$ as a (Maclaurin) series.

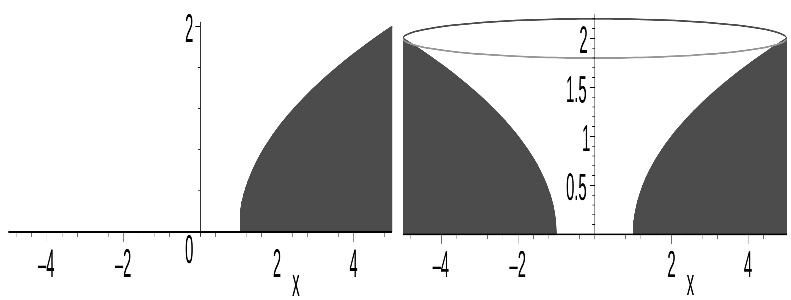
   (b) Using your answer to (a), write $g(x) = \arctan x$ as a (Maclaurin) series.

10. Use power series to solve the differential equation $y'' = -y$, $y(0) = 1$, $y'(0) = 0$. Can you identify the function y?

Answers

1. a) $xe^x - e^x$ b) $\frac{-1}{3}(2^{3/2} - 3^{3/2})$ c)12 d)$\frac{1}{2}$ e) $5x^4e^{-x^{10}}$

2. (a) Here are the area and the volume:



(b) $2\pi \int_1^5 x\sqrt{x-1}\,dx$

(c) $\pi \int_0^2 [5^2 - (y^2+1)^2]\,dy$

n	x_n	y_n	$y_{n+1} = y_n + y'(\Delta x)$
0	0	1	1+1(0.1)
1	0.1	1.1	1.1+1.2(0.1)
2	0.2	1.22	1.22+1.42(0.1)
3	0.3	1.362	1.362+1.662(0.1)
4	0.4	1.5282	done

3. $F = 62.4\pi(\frac{1}{2}y)^2\,dy$ and $d = 8 - y$ so $work = 62.4\pi \int_0^6 \frac{1}{4}(8y^2 - y^3)\,dy$

4. $y(x) = 2e^{x^2-x}$

5. $P(t) = \dfrac{M}{1 + Ae^{-kMt}}$ Limit is M.

6. a) Diverges b) $\frac{72}{7}$ c) e^3

7. $-1 \le x < 0$

8. (a) $\sum_{n=0}^{\infty} (-x)^2$

   (b) $\sum_{n=1}^{\infty} \dfrac{(-1)^n x^{2n+1}}{n}$

9. $\sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n}}{(2n)!}$ which is $y(x) = \cos x$