Exam 3 Name______________________________________

Final Exam Monday-April 21, 2003 10:00-11:50 CLAPP L9

1. Set up the integral for the arclength of the ellipse given by

$$x(t) = 4\cos t, \qquad y(t) = 9\sin t \qquad \text{for } \; 0 \le t \le 3.$$

2. Set up the equations to determine $(\overline{x}, \overline{y})$, the center of mass, centroid of the region bounded by $y = 10 - 2x$ and the x-axis on $2 \le x \le 5$.

3. State whether the series converges or diverges. Explain your reasoning by stating the test that would verify your answser.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ ________________________ test________________________

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2n-1}$ ________________________ test________________________

   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{2 \cdot 3^n}{4^n}$ ________________________ test ________________________

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{3n}$ ________________________ test________________________

4. Determine the Taylor Series for $f(x) = \dfrac{1}{1+x^2}$ for $-1 < x < 1$.

5. Determine the Taylor Series for $f(x) = \arctan x$ about $x = 0$ on $-1 < x < 1$.

6. Determine the Taylor Series for $h(x) = \sqrt{1+x^2}$ about $x = 0$ on $-1 < x < 1$.

7. Determine the Taylor Series for $f(x) = \cos x$ about $x = 0$. (show your work)

8. Determine the Taylor Series for $h(x) = \cos 2x$ about $x = 0$.

9. Determine the sum of the series.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} =$ ____________

(b) $\sum_{n=0}^{\infty} \frac{(-1)^{n}}{n!} =$ ____________

(c) $\sum_{n=1}^{\infty} 5\left(\frac{2}{7}\right)^{n} =$ ____________

10. Determine the interval of convergence of the series $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^2}$.