Exam III Name ________________________________

Final Exam:

Thursday 12/12 (2:00-3:50)

1. Solve the initial-value differential equation

$$x^2\, y' + y = 3 \qquad\qquad y(1) = 0.$$

2. Show that the circle of radius $r = 5$ has a circumference of $C = 10\pi$. The circle is defined by the parametric equations:

$$x(t) = 5\cos t, \qquad y(t) = 5\sin t \qquad\qquad \text{for } \ 0 \le t \le 2\pi.$$

3. Set up the equations for $(\overline{x}, \overline{y})$, the center of mass, centroid of the region bounded by $y = \cos(x) + \sin(x)$ for $0 \le x \le \pi/2$.

$\overline{x} =$ ________________________ $\qquad$ $\overline{y} =$ ________________________

4. State yes or no for convergence of the given series.

(a) $\sum_{n=0}^{\infty} \frac{n}{n^2+1}$ ____________

(d) $\sum_{n=1}^{\infty} \frac{1}{n^2+4}$ ____________

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$ ____________

(e) $\sum_{n=1}^{\infty} \frac{2^{3n}}{3^n}$ ____________

(c) $\sum_{n=0}^{\infty} \frac{3^n}{4^{n+2}}$ ____________

(f) $\sum_{n=1}^{\infty} \frac{1}{2n+1}$ ____________

5. Determine the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{x^n}{2^n n^2}$.

6. Determine the Taylor Series for $f(x) = \frac{1}{\sqrt{1-2x}}$ for $-1/2 < x < 1/2$ and show the first three terms of the series.

7. Determine the Taylor Series for $f(x) = \ln x$ about $x = 1$. (You must show your work.)

8. Determine the sum of the series.

(a) $\sum_{n=0}^{\infty} \left(\frac{2}{5}\right)^n$ $=$ ____________

(b) $\sum_{n=0}^{\infty} \frac{1}{n!}$ $=$ ____________

For 5 extra points, determine the values of $(\overline{x}, \overline{y})$ in problem 3.