Exam II Name _____

1. Integrate the following

(a) $\int 4xe^{-3x}\,dx$

answer: $-\frac{4}{3}xe^{-3x} - \frac{4}{9}e^{-3x} + c$

$u = 4x$ $\qquad dV = e^{-3x}dx$

$du = 4dx$ $\qquad V = -\frac{1}{3}e^{-3x}$

(b) $\int \cos^3 2x\,dx$

answer: $\frac{1}{2}\sin 2x - \frac{1}{6}\sin^3 2x + c$

$\int (1 - \sin^2 2x)\cos 2x dx = \int \left(cos2x - \sin^2 2x \cos 2x\right) dx$

(c) $\int \frac{dx}{\sqrt[3]{x} + x}$

answer: $\frac{3}{2}\ln|1 + x^{2/3}| + c$

Let $u = \sqrt[3]{x}$ then $u^3 = x$ and $3u^2\,du = dx$

$\int \frac{3u^2}{u + u^3}\,du = \int \frac{3u}{1 + u^2}\,du$

(d) $\int \sqrt{9-x^2}\,dx$

answer: $\frac{9}{2}\arcsin(x/3) + \frac{1}{2}x\sqrt{9-x^2} + c$

Let $\sqrt{9-x^2} = 3\cos\theta$ and $x = 3\sin\theta$ then $dx = 3\cos\theta\,d\theta$

$\int 9cos^2\theta\,d\theta = \int \left(\frac{9}{2} + \frac{9}{2}\cos 2\theta\right) d\theta = \frac{9\theta}{2} + \frac{9}{4}\sin 2\theta = \frac{9\theta}{2} + \frac{9}{2}\sin\theta\cos\theta$

(e) $\int \frac{x^2+4x-8}{x^2-5x+4}\,dx$

answer: $x + \ln|x-1| + 8\ln|x-4| + c$

divide first: $\int \left(x + \frac{9x-12}{x^2-5x+4}\right) dx = \int \left(x + \frac{9x-12}{(x-4)(x-1)}\right) dx$

2. Tell whether the integral converges or diverges. If it converges, give the limit. If it diverges, show why.

(a) $\int_2^\infty \frac{dx}{(x-1)^2}$

answer: $-\frac{1}{x-1}\Big|_2^{t\to\infty} = 0 - (-1) = 1$ converges

(b) $\int_2^{10} \frac{dx}{(x-2)^{3/2}}$

answer: $-\frac{2}{\sqrt{x-2}}\Big|_{t\to 2}^{10} = -\frac{2}{\sqrt{8}} + \frac{2}{\sqrt{0}} = \infty$ diverges

(c) $\int_0^\infty \frac{1}{1+9x^2}\,dx$

answer: $\frac{1}{3}\arctan(3x)\Big|_0^{t\to\infty} = \frac{1}{3}\cdot\frac{\pi}{2} = \frac{\pi}{6}$ converges

3. Solve the following differential equation for y.

(a) $y' - y^2 e^{2x} = 0$ $y(0) = 1.$

answer:

$$\frac{1}{y^2} dy = e^{2x} dx$$

$$-\frac{1}{y} = \frac{1}{2} e^{2x} + c$$

$$y = \frac{-2}{e^{2x} - 3}$$

(b) $y' + \frac{y}{10+t} = 6$ $y(0) = 2.$

answer:

$I(x) = e^{\int \frac{1}{10+t} dt} = e^{\ln(10+t)} = 10 + t$

$((10+t)y)' = 6(10+t)$

$y(t) = \frac{3t^2 + 60t + 20}{10+t}$

4. Suppose your bank offers 3% continual annual interest on the balance of your certificate of deposit but you make a continual withdrawal of \$300 per year.

(a) Write a differential equation giving the balance of your CD at any time t. Let $B(t)$=balance at time t.
answer: $\frac{dB}{dt} = .03B - 300$

(b) Solve this differential equation and give the minimum initial (and only) deposit so that your funds are not depleted over time.
answer: $B(t) = 10,000 + Ae^{0.03t}$
minimum initial deposit $\$10,000$

(c) Determine $B(t)$ if you initially deposit $\$12,000$ and do not depoit again.
answer: $B(t) = 10,000 + 2,000e^{0.03t}$