1. Integrate the following

(a) \[ \int 4xe^{-3x} \, dx \]
   \[
   \text{answer: } -\frac{4}{3}xe^{-3x} - \frac{4}{9}e^{-3x} + c
   \]
   
   \[ u = 4x \quad dV = e^{-3x} \, dx \]
   \[ du = 4dx \quad V = -\frac{1}{3}e^{-3x} \]

(b) \[ \int \cos^3 2x \, dx \]
   \[
   \text{answer: } \frac{1}{2} \sin 2x - \frac{1}{6} \sin^3 2x + c
   \]
   
   \[ \int (1 - \sin^2 2x) \cos 2x \, dx = \int (\cos 2x - \sin^2 2x \cos 2x) \, dx \]

(c) \[ \int \frac{dx}{\sqrt{x} + x} \]
   \[
   \text{answer: } \frac{3}{2} \ln |1 + x^{2/3}| + c
   \]
   
   Let \( u = \sqrt{x} \) then \( u^3 = x \) and \( 3u^2 \, du = dx \)
   
   \[ \int \frac{3u^2}{u + u^3} \, du = \int \frac{3u}{1 + u^2} \, du \]
(d) \[ \int \sqrt{9-x^2} \, dx \]
answer: \[ \frac{9}{2} \arcsin \left( \frac{x}{3} \right) + \frac{1}{2} x \sqrt{9-x^2} + c \]

Let \( \sqrt{9-x^2} = 3 \cos \theta \) and \( x = 3 \sin \theta \) then \( dx = 3 \cos \theta \, d\theta \)
\[ \int 9\cos^2 \theta \, d\theta = \int \left( \frac{9}{2} + \frac{9}{4} \sin 2\theta \right) \, d\theta = \frac{9\theta}{2} + \frac{9}{2} \sin 2\theta = \frac{9\theta}{2} + \frac{9}{2} \sin \theta \cos \theta \]

(e) \[ \int \frac{x^2 + 4x - 8}{x^2 - 5x + 4} \, dx \]
answer: \[ x + \ln |x - 1| + 8 \ln |x - 4| + c \]

divide first: \[ \int \left( x + \frac{9x - 12}{x^2 - 5x + 4} \right) \, dx = \int \left( x + \frac{9x - 12}{(x - 4)(x - 1)} \right) \, dx \]

2. Tell whether the integral converges or diverges. If it converges, give the limit. If it diverges, show why.

(a) \[ \int_2^\infty \frac{dx}{(x - 1)^2} \]
answer: \[ \left. -\frac{1}{x - 1} \right|_{2}^{t\to\infty} = 0 - (-1) = 1 \text{ converges} \]

(b) \[ \int_{2}^{10} \frac{dx}{(x - 2)^{3/2}} \]
answer: \[ \left. -\frac{2}{\sqrt{x - 2}} \right|_{2}^{10} = -\frac{2}{\sqrt{8}} + \frac{2}{\sqrt{0}} = \infty \text{ diverges} \]

(c) \[ \int_{0}^{\infty} \frac{1}{1 + 9x^2} \, dx \]
answer: \[ \left. \frac{1}{3} \arctan(3x) \right|_{0}^{t\to\infty} = \frac{1}{3} \cdot \frac{\pi}{2} = \frac{\pi}{6} \text{ converges} \]
3. Solve the following differential equation for $y$.

(a) \[ y' - y^2e^{2x} = 0 \quad y(0) = 1. \]

answer:
\[
\frac{1}{y^2}dy = e^{2x}dx \\
\frac{1}{y} = \frac{1}{2}e^{2x} + c \\
y = \frac{-2}{e^{2x} - 3}
\]

(b) \[ y' + \frac{y}{10 + t} = 6 \quad y(0) = 2. \]

answer:
\[
I(x) = e^{\int \frac{1}{10+t}dt} = e^{\ln(10+t)} = 10 + t \\
((10 + t)y)' = 6(10 + t) \\
y(t) = \frac{3t^2 + 60t + 20}{10 + t}
\]

4. Suppose your bank offers 3% continual annual interest on the balance of your certificate of deposit but you make a continual withdrawal of $300 per year.

(a) Write a differential equation giving the balance of your CD at any time $t$. Let $B(t)$ = balance at time $t$.

answer: \[ \frac{dB}{dt} = .03B - 300 \]

(b) Solve this differential equation and give the minimum initial (and only) deposit so that your funds are not depleted over time.

answer: $B(t) = 10,000 + Ae^{0.03t}$

minimum initial deposit $10,000$

(c) Determine $B(t)$ if you initially deposit $12,000 and do not deposit again.

answer: $B(t) = 10,000 + 2,000e^{0.03t}$