OPTIMIZATION

1. A rectangular region is to be fenced using 4800 feet of fencing. If the rectangular region is to be separated into 3 regions by running two lines of fence parallel to two opposite sides, determine the dimensions of the region which maximizes the area of the region.

answer: (1) Maximize area of rectangle under constraint total fence=4800
(2) \( A = xy \)
(3) \( A = (2400 - 2y)y \)
\( A = 2400y - 2y^2 \)
(4)

\[
A'(y) = 2400 - 4y \\
0 = 2400 - 4y \\
y = 600 \\
x = 1200
\]

2. A box with a square base, rectangular sides and open top is to contain 12 cubic feet of space. If the material for its base costs $3/ft^2 and that for its sides costs $1/ft^2, determine the dimensions of the box so that the cost of the material is a minimum.

answer: (1) Minimize cost under the constraint volume=12
(2) \( C = 3x^2 + 4xy \)
(3) \( C = 3x^2 + 4x \left( \frac{12}{x^2} \right) \)
\( C(x) = 3x^2 + \frac{48}{x} \)
(4)

\[
C'(x) = 6x - \frac{48}{x^2} \\
0 = 6x - \frac{48}{x^2} \\
\frac{48}{x^2} = 6x \\
x^3 = 8 \\
x = 2 \\
y = 3
\]
3. Determine the point(s) on the hyperbola \( x^2 - y^2 = 4 \) which are closest to the point \( (0, 8) \).

answer: (1) Minimize distance to \((0, 8)\) under constraint \((x, y)\) lies on hyperbola.

\[
(2) \quad D = \sqrt{x^2 + (y - 8)^2} \quad \text{subject to} \quad x^2 - y^2 = 4
\]

\[
(3) \quad D = \sqrt{4 + y^2 + (y - 8)^2}
\]

\[
D'(y) = \frac{2y + 2(y - 8)}{\sqrt{4 + y^2 + (y - 8)^2}}
\]

\[
0 = \frac{2y + 2(y - 8)}{\sqrt{4 + y^2 + (y - 8)^2}}
\]

\[
0 = 2y - 16
\]

\[
y = 4
\]

\[
x = \pm \sqrt{20}
\]

4. If 1200 cm\(^2\) of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

answer: (1) Maximize \textit{volume} under constraint \textit{material}=1200 cm\(^2\)

\[
(2) \quad V = x^2y \quad \text{subject to} \quad x^2 + 4xy = 1200
\]

\[
(3) \quad V = x^2 \left( \frac{1200 - x^2}{4x} \right)
\]

\[
V = 300x - \frac{1}{4}x^3
\]

\[
V'(x) = 300 - \frac{3}{4}x^2
\]

\[
0 = 300 - \frac{3}{4}x^2
\]

\[
x^2 = 400
\]

\[
x = 20
\]

\[
y = 10
\]

\[
V = 4000
\]