1. (a) Find the derivative of the function

\[ f(x) = \frac{1}{x} + 2\cos x + 3\tan x + 4\cot x + 5\ln x + 6e^x + 7x^8 + 9\arctan x + \arcsin x \]

(b) Let \( f(x) = x^2 \). Find \( f'(2) \) by using only the definition of the derivative.

(c) Geometrically, the definite integral \( \int_a^b f(x) \, dx \) represents the area of a certain region on the \( x - y \) plane that is related to the curve \( y = f(x) \).

Using only this geometrical interpretation find \( \int_0^3 (2x + 1) \, dx \).

2. Let \( \vec{a} = \langle 2, 1 \rangle \) and \( \vec{b} = \langle 1, 3 \rangle \)

(a) Find the angle between \( \vec{a} \) and \( \vec{b} \)

(b) Let \( P = \langle 1, 3 \rangle \), \( Q = \langle -1, 5 \rangle \) and \( S = \langle 5, 7 \rangle \) be points in a plane. Find the fourth vertex of the parallelogram whose sides are \( P\vec{Q} \) and \( P\vec{S} \).

3. The position vector of a particle traveling on the \( x - y \) plane at time \( t \) is \( \vec{r}(t) = \langle t, 8t - t^2 \rangle \), where \( t \) is measured in seconds and coordinates are in meters.

(a) Find the particle’s average velocity vector during the time interval \([0, 2]\).

(b) Find the particle’s velocity vector, speed, and acceleration vector at time \( t = 1 \).

(c) Find a non-parametric equation describing the curve that the particle passes by.

4. Find the derivatives of the following functions.

(a) \( f(x) = (x^2 + x + 1)(x^3 - 3x^2 + x + 1) \). [no simplification for answer]

(b) \( g(x) = \frac{x + 1}{x^2 + 1} \).

(c) \( p(x) = (1 + x^4)^{10} \)

(d) \( q(x) = \sin \left( \frac{1}{xe^{2x}} \right) \)

5. (a) Evaluate \( \lim_{h \to 0} \frac{(x + h)^2 - x^2}{h} \)
Evaluate \[ \lim_{x \to 0} \frac{1 - e^x}{\sin x} \]

Evaluate \[ \lim_{x \to 1} \frac{x - 1}{|x - 1|} \]

Evaluate \[ \lim_{x \to 0} \left( \frac{1 + x}{x \cos x} - \frac{1}{x} \right) \]

6. (a) Find the Riemann sum \( R_4 = \sum_{i=1}^{4} 4 f(c_i)(x_i - x_{i-1}) \) for \( \int_0^8 x^2 \, dx \) with regular partition points \( x_i = 2i \) for \( i = 0, 1, 2, 3, 4 \), and the middle point rule: \( c_i = \frac{1}{2}(x_{i-1} + x_i) \).

(b) Evaluate the definite integral \( \int_0^8 x^2 \, dx \).

(c) Evaluate the indefinite integral

\[
\int \left( x^2 + \frac{2}{x} + 3 \cos x + \frac{4}{\sqrt{1 - x^2}} + \frac{5}{1 + x^2} \right) \, dx
\]

(d) Find the derivative of the function \( F(x) = \int_0^x t^2 e^{t^2} \, dt \).

7. (a) Use a linear approximation or a differential for the function \( f(x) = x^{1/3} \) at \( a = 1000 \) to find an approximation to \( \sqrt[3]{1003} - \sqrt[3]{1000} \).

(b) Let Use the Newton’s Method to find a rational number that approximates the positive root to \( x^2 - 2 = 0 \).

8. The derivatives of the function \( f(x) = xe^{-x^2/2} \) are calculated as follows

\[ f'(x) = (1 - x^2)e^{-x^2/2}, \quad f''(x) = x(x^2 - 3)e^{-x^2/2} \]

(a) Find the intervals where \( f \) is increasing or decreasing. Also find points of local or global minimum or maximum.

(b) Find intervals where \( f \) is concave up or concave down. Also find points of inflection.
(c) Find any horizontal asymptotes.

(d) Sketch the curve of \( y = f(x) \) for \(-\infty < x < \infty\).

9. A box with a square base, rectangular sides, and open top must have a volume of 1000 cm\(^3\). The material for the base costs $4/cm\(^2\) and that for the sides $2/cm\(^2\). Find the dimensions of a box that minimizes the cost of material used.

10. A swimming pool of dimension 100(m) \( \times \) 200(m) and horizontal bottom is drained at a rate of 2 m\(^3\)/min. Find the rate of decreasing of the depth of the water in the pool.

11. (a) Let \( q(x) = x^2 \). Using the logarithmic differentiation technique, find \( q'(x) \).

(b) Let \( y = y(x) \) be implicitly defined by \( y^3 + xy = 1 \). Using the implicit differentiation technique, find \( y'(x) \).

(c) Find the equation of the line that has slope 3 and is tangent to the curve given parametrically by \( x = t^2 + 1, \ y = t^3 \).