Math 0220 Sample Final 2

1. Let $\vec{a} = \langle 2, 1 \rangle$ and $\vec{b} = \langle 1, 3 \rangle$.

(3 pts.)

1a. Evaluate $|\vec{a} + \vec{b}|$

(3 pts.)

1b. Find the unit vector in the direction of $\vec{b}$.

(4 pts.)

1c. Find all values of $t$ such that $\vec{a}$ is perpendicular to $\vec{c} = \langle -4, 8t \rangle$. 
2a. Give a parametric vector equation for a circle of radius 9 with the center at the point $(1, -2)$.

2b. The trajectory of an object is determined by

$$\mathbf{r}(t) = \langle 2t, -2t^2 + 16t \rangle$$

where $-\infty < t < \infty$.

Eliminate the parameter $t$ and find an equation in $x$ and $y$ that describes the curve on which the object moves.
3. Let \( f(x) = x(x - 1)^2, -\infty < x < \infty. \)

(10 pts.)

3a. Find all points where \( f \) has a local maximum or local minimum. Justify your answers.

(10 pts.)

3b. Find all inflection points. Justify your answer.

(10 pts.)

3c. Graph the function.
(10 pts.)

4. Find $x_2$, the second iterate in Newton’s method, to find an approximate value for the negative solution of $x^4 = 10100$. Assume that $x_1 = -10$. Show all details.
(10 pts.)

5a. Find the equation for the line tangent to $y = x^{1/4}$ at $x = 10000$. **Hint:** $(10000)^{1/4} = 10$.

(5 pts.)

5b. Use the tangent line found in part (a) to obtain an approximate value for $(10100)^{1/4}$. 
6a. Evaluate: \[ \lim_{x \to 0} \frac{x}{\sqrt{x + 4} - 2} \]

6b. Evaluate: \[ \lim_{x \to 3^-} \frac{|x - 3|}{x - 3} \]

6c. Evaluate: \[ \lim_{x \to 1} \frac{\arctan(\tan(2x - 3))}{2x - 5} \]

6d. Evaluate: \[ \lim_{x \to -\infty} \frac{\ln(1 + \frac{3}{x})}{\sin\left(\frac{4}{x^2}\right)} \]

6e. Evaluate: \[ \lim_{x \to 0} x^2 \ln(x^2) \]
(10 pts.)

7a. Evaluate: \( \int \frac{dx}{4 + 25x^2} \)

(10 pts.)

7b. Evaluate: \( \int (12x + x^{1/2})dx \)

(10 pts.)

7c. Let \( f(x) = \int_{0}^{2x} \frac{dt}{\sqrt{1 + t^2}} \). Find \( \frac{df}{dx} \).
(10 pts.)

8a. Find $\frac{dy}{dx}$ at the point $(x, y) = (0, 1)$ on the curve defined by the equation $y^2 + x e^{y^2} = 1$.

(10 pts.)

8b. Let $y = \arctan(3 \sin^2(x))$. Find $y' \left( \frac{\pi}{4} \right)$.

(10 pts.)

8c. Let $y = x^{2x}$. Find $\frac{dy}{dx}$. 
9. A particle moves along the curve $2x^2 - xy + 3y^2 = 24$. If at a given time, the particle is at position $(-3, 1)$ and the $x$ coordinate of its velocity at this point is 5 then find the $y$ coordinate of the velocity.