Math 0220 Sample Final 1

(10 pts.)

1a. A particle moves with speed 2 around a circle of radius 4 centered at \((x, y) = (1, 0)\). Assume that the particle is at \((x, y) = (5, 0)\) at time \(t = 0\). Find the vector equation describing the motion of the particle if it moves clockwise around the circle as \(t\) increases.

(15 pts.)

1b. The trajectory of an object is described by the vector function

\[
\vec{r} = (4 + 7t^3)i + (1 - 2t)j, \quad -\infty < t < \infty
\]

Eliminate \(t\) and find an equation in \(x\) and \(y\) that describes the curve on which the object moves.
2. Use a tangent line to the function \( f(x) = (8x)^{1/3} \) to find an approximate value for \((8.08)^{1/3}\).

(10 pts.)

3. Using Newton’s method, find \( x_2 \), the second iterate, to approximate the solution of \( x^5 + x^3 = 1 \). Assume that \( x_1 = 1 \).
4. Given the function:

\[ f(x) = \begin{cases} 
\frac{1}{x^2}, & -\infty < x \leq -1 \\
-x, & -1 < x \leq 0 \\
\frac{x}{2}, & 0 < x \leq 1 \\
1+x^2, & x > 0 
\end{cases} \]

Determine:

(5 pts. each)

4a. \( \lim_{x \to -1^+} f(x) \)

4b. \( \lim_{x \to 0^-} f'(x) \)

4c. Sketch the graph of the function.
(6 pts. each)

5. Find the first derivative of the following functions:

5a. \[ f(x) = \tan^{-1}(x^3 + 2x) \]

5b. \[ s(x) = \sin^2(x) - \frac{3}{x^{1/3}}, \quad x \neq 0. \]

5c. \[ y = \frac{x}{\ln(x)}. \]

5d. \[ h(x) = 3^{\tan(x)}. \]

5e. \[ y = x^3 \ln(x^2) \]
6. Determine the following limits:

6a. \( \lim_{h \to 0^+} \frac{|-2 + h| - |-2|}{h} \)

6b. \( \lim_{x \to 0} \frac{\tan^{-1}(2 + x) - \tan^{-1}(2)}{x} \)

6c. \( \lim_{x \to 0^+} x^2 \ln(x^3) \)

6d. \( \lim_{t \to 0} (1 + 3t)^{\frac{1}{2}} \)

6e. \( \lim_{x \to 0} x^2(1 - \cos(2x))^{-1} \)
(15 pts.)

7. Find the equation for the line tangent to the graph of the equation \( \sqrt{y+x} - \sqrt{y-x} = 2 \) at \( Q = (10, 26) \).
8. A spherically shaped balloon is being inflated by pumped air. The area of its surface is $S = 4\pi r^2$ square inches, and its volume is $V = \frac{4}{3}\pi r^3$ cubic inches, where $r$ is the radial distance from the center of the balloon to its surface. As air is pumped into the balloon, assume that the area of the surface is increasing at a rate of 8 square inches per second. How fast is its radius increasing when the volume reaches $\frac{32\pi}{3}$ cubic inches.
9. A wire 16 feet long has to be formed into a rectangle. What dimensions should the rectangle have to maximize its area?
10a. Find the area under the curve: \( y = 2^x \) between \( x = 0 \) and \( x = 5 \).

10b. Evaluate \( \int \frac{(x+1)}{1+2x^2} \, dx \)
11. Consider the function \( f = x^2 e^{-x} \) where \(-\infty < x < \infty\).

(7 pts.)

11a. Find all values of \( x \) where \( f \) attains a relative maximum or a relative minimum. Justify your answer.

(3 pts.)

11b. Sketch the graph of the function.