Program

All talks are in *David Lawrence Hall room 105* on the main campus of University of Pittsburgh.

**Sponsors:** Mathematics Research Center (MRC) of the University of Pittsburgh.

**Organizers:** Ming Chen and Anna Vainchtein.
Nonlinear Waves in Discrete and Continuum Systems  
University of Pittsburgh, June 17-18, 2022

*All talks are in Lawrence Hall room 105.*

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Friday, June 17

✦ **Morning Session**

- **08:40-09:30**: Alexandru Ionescu, Princeton University  
  *On the long-term regularity of water waves in large periodic domains*

- **09:40-10:30**: Guillaume James, INRIA-Grenoble  
  *Impact propagation in granular chains*

10:30-11:00: Coffee Break

- **11:00-11:50**: Dmitry Pelinovsky, McMaster University  
  *KP-II approximation in dynamics of two-dimensional FPU systems*

12:00-02:00pm: Lunch Break

✦ **Afternoon Session**

- **02:00-02:50**: Mark Ablowitz, University of Colorado at Boulder  
  *Integrable fractional and nonlocal nonlinear evolution equations*

- **03:00-03:50**: Samuel Walsh, University of Missouri  
  *Orbital stability of internal waves*

03:50-04:20pm: Coffee Break

- **04:20-05:10**: Miles Wheeler, University of Bath  
  *Overhanging water waves with constant vorticity*
Saturday, June 18

**MORNING SESSION**

- **08:40-09:30:** Panayotis Kevrekidis, University of Massachusetts at Amherst  
  *On some select Klein-Gordon and beam problems: internal modes, fat tails, wave collisions and beyond*

- **09:40-10:30:** Michael Herrmann, TU Braunschweig  
  *Integral equations for waves in discrete and continuum media*

10:30-11:00: Coffee Break ☕

- **11:00-11:50:** Barbara Prinari, State University of New York at Buffalo  
  *Inverse Scattering Transform, solitons and soliton interactions for the complex coupled short-pulse equation*

12:00-02:00pm: Lunch Break 🍽️

**AFTERNOON SESSION**

- **02:00-02:50:** Marcelo Disconzi, Vanderbilt University  
  *General-relativistic viscous fluids*

- **03:00-03:50:** Jeremy Marzuola, University of North Carolina  
  *Degenerate dispersive equations*

03:50-04:20pm: Coffee Break ☕

- **04:20-05:10:** J. Douglas Wright, Drexel University  
  *A KdV approximation for random FPUT lattices*

THE END. 😊
Abstracts

Mark Ablowitz, University of Colorado at Boulder
Title: Integrable fractional and nonlocal nonlinear evolution equations
Abstract: In recent years fractional evolution equations have been heavily studied. A class of integrable nonlinear fractional wave equations can be integrated by the Inverse Scattering Transform (IST). Paradigms include the integrable fractional Korteweg-deVries and nonlinear Schrödinger equations. If time permits a novel class of integrable nonlocal nonlinear equations and soliton solutions will also be discussed.

Marcelo Disconzi, Vanderbilt University
Title: General-relativistic viscous fluids
Abstract: In this talk, we will review some recent developments at the intersection of mathematics and physics regarding theories of relativistic fluids with viscosity. The discovery of the quark-gluon plasma that forms in heavy-ion collision experiments provides a unique opportunity to study properties of matter under extreme conditions, as the quark-gluon plasma is the hottest, smallest, and densest fluid known to humanity. Studying the quark-gluon plasma also provides a window into the earliest moments of the universe, since microseconds after the Big Bang the universe was filled with matter in the form of the quark-gluon plasma. For more than two decades, the community has intensely studied the quark-gluon plasma with the help of a rich interaction between experiments, theory, phenomenology, and numerical simulations. From these investigations, a coherent picture has emerged, indicating that the quark-gluon plasma behaves essentially like a relativistic liquid with viscosity. More recently, state-of-the-art numerical simulations strongly suggested that viscous and dissipative effects can also have non-negligible effects on gravitational waves produced by binary neutron star mergers. But despite the importance of viscous effects for the study of such systems, a robust and mathematically sound theory of relativistic fluids with viscosity is still lacking. This is due, in part, to difficulties in preserving causality upon the inclusion of viscous and dissipative effects into theories of relativistic fluids. In this talk, we will survey the history of the problem and report on a new approach to relativistic viscous fluids that addresses these issues.

Michael Herrmann, TU Braunschweig
Title: Integral equations for waves in discrete and continuum media
Abstract: Coherent structures in Hamiltonian or dissipative systems can often be regarded as the eigenfunctions of a nonlinear convolution operator, but the corresponding spectral theory is much more complicated than its linear counterpart. In this talk, we present typical examples and sketch some of the known existence and approximation results. We also discuss certain asymptotic regimes and the difficulties in the uniqueness problem.
(joint work with Karsten Matthies, University of Bath)
Alexandru Ionescu, Princeton University

Title: On the long-term regularity of water waves in large periodic domains

Abstract: I will talk about some recent work in collaboration with Yu Deng and Fabio Pusateri on the long term regularity of two-dimensional gravity water waves in a large periodic box of size $R$. We consider small data in Sobolev spaces and prove long-term regularity both in the deterministic and in the random case.

Guillaume James, INRIA-Grenoble

Title: Impact propagation in granular chains

Abstract: Impacts on chains of identical beads can generate nonlinear waves such as solitary waves or fronts. We consider a long-wave regime that occurs when the exponent of the contact force approaches unity. In the absence of dissipation, solitary wave profiles can be approximated by a Gaussian solution to a logarithmic KdV equation. When a small contact damping is introduced, the analogous continuum limit corresponds to a logarithmic KdV-Burgers equation. This equation admits traveling front solutions which approximate compression fronts in the granular chain. We validate this approximation numerically, using both dynamical simulations (response of the chain to a compression by a piston) and the Newton method (computation of exact traveling waves by a shooting method).

Panayotis Kevrekidis, University of Massachusetts at Amherst

Title: On some select Klein-Gordon and beam problems: internal modes, fat tails, wave collisions and beyond

Abstract: In this talk, we revisit some very thoroughly studied (yet still quite entertaining!) problems involving kink-like structures in non-integrable systems starting with the Klein-Gordon models. We will start from the prototypical model of the $\phi^4$ class and discuss a bit its history of discoveries, successes and culprits as concerns the complex landscape of collisions of kinks and antikinks and its fractal features. We will briefly touch upon the recent developments in this vein, as well as the remarkable feature that after about 50 years of studies, there are still some fundamental questions remaining. We will then extend considerations to the case of higher order ($\phi^{6,8,10}$ and $12$) models and present the particularities that each of these models bears, including the potential for numerous internal modes, fat tails and power law kink-antikink interactions among others. Time permitting, we ’ll also open up the problem towards the possibility of higher order dispersion and motivate such considerations from the perspective of recent optics problems. We will see how such higher order dispersion also creates interesting possibilities such as oscillatory tails, numerous kink-antikink bound states and their own complex interaction landscapes.

Jeremy Marzuola, University of North Carolina

Title: Degenerate dispersive equations

Abstract: We discuss recent work on some quasilinear toy models for the phenomenon of degenerate dispersion, where the dispersion relation may degenerate at a point in physical space. In particular, we discuss the existence and uniqueness of solutions for degenerate KdV and NLS equations. This is joint work with Pierre Germain and Ben
Harrop-Griffiths. We will also discuss a discrete model that was the motivation for this work and recent progress on Gibbs Measures for such equations with Jonathan Mattingly and Dana Mendelson.

Dmitry Pelinovsky, McMaster University

Title: KP-II approximation in dynamics of two-dimensional FPU systems

Abstract: Dynamics of the Fermi–Pasta–Ulam (FPU) system on a two-dimensional square lattice is considered in the limit of small-amplitude long-scale waves with slow transverse modulations. In the absence of transverse modulations, dynamics of such waves, even at an oblique angle with respect to the square lattice, is known to be described by the Korteweg–de Vries (KdV) equation. We prove that the transversely modulated waves are well described by the Kadomtsev–Petviashvili (KP-II) equation. The result was expected long ago but proving rigorous bounds on the approximation error turns out to be complicated due to the nonlocal terms of the KP-II equation. The result holds for the propagating waves at every angle with respect to the square lattice.

Barbara Prinari, State University of New York at Buffalo

Title: Inverse Scattering Transform, solitons and soliton interactions for the complex coupled short-pulse equation

Abstract: We present the inverse scattering transform (IST) for the complex coupled short pulse equation (ccSPE) on the line. Our work extends to the complex, coupled case the Riemann- Hilbert approach to the IST for the real, scalar short-pulse equation proposed by A. Boutet de Monvel and collaborators in 2016. One-soliton solutions are also investigated within the framework of the IST. The simplest soliton solutions, fundamental solitons, are found to be the natural vector generalization of scalar one-soliton solutions of the complex short-pulse equation. But in the coupled case one can also have more complicated, composite soliton solutions, corresponding to two fundamental solitons having the same amplitude and velocity but different carrier frequencies, as well as solutions that, while still corresponding to a minimal set of discrete eigenvalues, cannot be reduced to simple superposition of fundamental solitons. Moreover, it is found that the same constraint on the discrete eigenvalues which leads to regular, smooth one-soliton solutions in the complex SPE, also holds in the coupled case, for both a single fundamental soliton and a single fundamental breather, but not, in general, in the case of a composite breather. If time permits, the interactions of fundamental solitons will be discussed.

Samuel Walsh, University of Missouri

Title: Orbital stability of internal waves

Abstract: In this talk, we discuss recent work on the nonlinear stability of capillary-gravity waves propagating along the interface dividing two immiscible fluid layers of finite depth. The motion in both regions is governed by the incompressible and irrotational Euler equations, with the density of each fluid being constant but distinct. A diverse collection of small-amplitude solitary wave solutions for this system have been constructed in the case of strong surface tension (as measured by the Bond number) and slightly subcritical Froude number. We prove that all of these waves are (conditionally) orbitally stable in the natural energy space. Moreover, the trivial solution is shown to be conditionally stable when the Bond and Froude numbers lie in a certain unbounded parameter region. For
the near critical surface tension regime, we prove that one can infer conditional orbital stability or orbital instability of small-amplitude traveling waves solutions to the full Euler system from considerations of a dispersive PDE model equation.

These results are obtained by reformulating the problem as an infinite-dimensional Hamiltonian system, then applying a version of the Grillakis–Shatah–Strauss method developed with Varholm and Wahlén. This is joint work with Robin Ming Chen.

Miles Wheeler, University of Bath

Title: Overhanging water waves with constant vorticity

Abstract: We consider the steady water wave problem in two dimensions. Without vorticity or surface tension, it is known that the surface of the fluid must be a graph. With nonzero constant vorticity, on the other hand, numerics have long shown so-called ‘overhanging’ waves. I will present the first rigorous construction of such waves, obtained by perturbing a new family of explicit solutions to the problem in the zero-gravity limit. Interestingly, these explicit solutions have the same surfaces as Crapper’s celebrated irrotational capillary waves, even through the flow beneath the surface is completely differently. In the second part of the talk, I will present global bifurcation results for solitary waves with constant vorticity. Here the main novelty is the formulation of the problem as an elliptic system for two real-valued functions, one describing the conformal mapping of the fluid domain and another describing the motion inside the fluid. This is joint work with Vera Hur and with Susanna Haziot.

J. Douglas Wright, Drexel University

Title: A KdV approximation for random FPUT lattices

Abstract: We consider a Fermi-Pasta-Ulam-Tsingou lattice with randomly varying coefficients. Under some technical conditions on the nature of the randomness we are able to prove that small amplitude/long wavelength solutions are rigorously approximated by solution of Korteweg-de Vries equation for very long times. The key ideas follow from stochastic homogenization theory and rely heavily on classical but deep results about random walks. This work is joint with Joshua McGinnis.