1. (10 pts.) Let $f(x) = x^2$.

(a) Approximate the area under the curve $y = f(x)$ from **a = -4 to b = 4** using a Riemann sum with 4 left rectangles. (Write the sum; you need not evaluate it.)

$$\Delta x = \frac{b-a}{n} = \frac{4-(-4)}{4} = 2$$

$$x_1 = -4,\ x_2 = -2,\ x_3 = 0,\ x_4 = 2$$

$$A \approx f(-4)2 + f(-2)2 + f(0)2 + f(2)2$$

$$A \approx 16\cdot 2 + 4\cdot 2 + 0\cdot 2 + 4\cdot 2 \left(= 32 + 8 + 8 = 48\right)$$

(b) Find the exact value of the area under the curve $y = f(x)$ from **a = -1 to b = 1** by evaluating an appropriate definite integral using the Fundamental Theorem of Integral Calculus.

$$A = \int_{-1}^{1} x^2\,dx = \frac{x^3}{3}\Big|_{-1}^{1} = \frac{1}{3}(1+1) = \frac{2}{3}$$

2. (15 pts.) Find the following integrals:

(a) $\int (x^3 - \sqrt[3]{x^5} + \frac{2}{x} + e^{-2x} + 8)dx = \frac{x^4}{4} - \frac{3}{8}x^{\frac{8}{3}} + 2\ln|x| - \frac{1}{2}e^{-2x} + 8x + C$

(b) $\int \frac{(x-1)^2}{x}dx = \int \frac{x^2-2x+1}{x}\,dx = \int \left(x-2+\frac{1}{x}\right)dx = \frac{x^2}{2} - 2x + \ln|x| + C$

(c) $\int_{-2}^{-1} \frac{1}{x}dx = \ln|x|\Big|_{-2}^{-1} = \ln|-1| - \ln|-2| = \ln(1) - \ln(2) = -\ln(2).$

3. (21 pts.) Use substitution to find the following integrals:

(a) $\int (e^x - 1)(e^x - x)^7\,dx = \int u^7\,du = \frac{u^8}{8} + c = \frac{(e^x - x)^8}{8} + c$

Let $u = e^x - x$

$du = (e^x - 1)\,dx.$

(b) $\int \frac{x^2+1}{x^3+3x}\,dx = \frac{1}{3}\int \frac{du}{u} = \frac{1}{3}\ln|u| + c = \frac{1}{3}\ln|x^3+3x| + C$

Let $u = x^3 + 3x$

$du = (3x^2+3)\,dx. = 3(x^2+1)\,dx$

(c) $\int_0^2 \frac{x^2}{\sqrt{x^3+1}}\,dx = \frac{1}{3}\int_1^9 \frac{du}{\sqrt{u}}. = \frac{1}{3}\cdot 2u^{\frac{1}{2}}\Big|_1^9 = \frac{2}{3}(3-1) = \frac{4}{3}$

Let $u = x^3 + 1$

$du = 3x^2\,dx.$

4. (7 pts.) A company's marginal cost function is $MC = 12x^2 + 3e^{-0.1x}$ where x is the number of units, and fixed costs are \$100. Find the cost function, C(x).

$C(x) = 4x^3 - 30e^{-0.1x} + K.$

$C(0) = -30 + K = 100$

$K = 130$

$C(x) = 4x^3 - 30e^{-0.1x} + 130$

5. (6 pts.) Find the average value of $f(x) = \frac{1}{x^3}$ on [1,3].

$f_{avg} = \frac{1}{3-1}\int_1^3 \frac{1}{x^3}\,dx = \frac{1}{2}\left(-\frac{1}{2}\right)x^{-2}\Big|_1^3 = -\frac{1}{4}\left[\frac{1}{9} - 1\right] = \frac{8}{36}$

$f_{avg} = \frac{2}{9}.$

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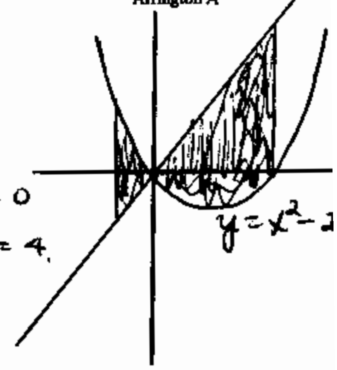
6. (14 pts.) Set up, but do not evaluate, integrals for the area

(a) Between the curves $y = x^2 - 2x$ and $y = 2x$ on [-1,2].

$$x^2-2x=2x$$
$$x^2-4x=0$$
$$x(x-4)=0$$
$$x=0,\ x=4.$$

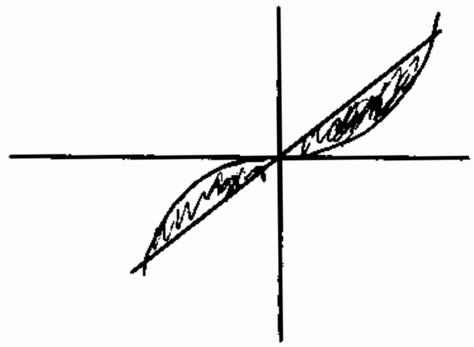


$$A=\int_{-1}^{0}(x^2-4x)\,dx + \int_{0}^{2}(4x-x^2)\,dx.$$

(b) Bounded by the curves $y = x^3$ and $y = x$.

$$x^3=x.$$
$$x^3-x = x(x^2-1) = x(x+1)(x-1)=0$$
$$x=-1,\ 0,\ 1$$

$$A=\int_{-1}^{0}(x^3-x)\,dx + \int_{0}^{1}(x-x^3)\,dx.$$



7. (6 pts.) Set up, but do not evaluate, a definite integral for the consumer's surplus for a demand function $d(x) = 200 - 30x$ dollars at the demand level $x = 5$.

$$d(5) = 200-150=50$$

$$C.S. = \int_{0}^{5}[(200-30x)-50]\,dx = \int_{0}^{5}(150-30x)\,dx$$

8. (21 pts.) Use integration by parts to find:

(a) $\int (x \ln x)\, dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int x\, dx = \frac{x^2}{2}\ln x - \frac{1}{4}x^2 + C.$

$u = \ln x \qquad dv = x\, dx$

$du = \frac{dx}{x} \qquad v = \frac{x^2}{2}.$

(b) $\int xe^{-x}\, dx = -xe^{-x} + \int e^{-x}\, dx = -xe^{-x} - e^{-x} + C$

$u = x \qquad dv = e^{-x}\, dx$

$du = dx \qquad v = -e^{-x}$

(c) $\int \frac{x}{\sqrt{x-3}}\, dx = 2x\sqrt{x-3} - 2\int \sqrt{x-3}\, dx = 2x\sqrt{x-3} - 2\cdot\frac{2}{3}(x-3)^{\frac{3}{2}} + C$

$u = x \qquad dv = \frac{dx}{\sqrt{x-3}}$

$du = dx \qquad v = 2\sqrt{x-3}$

(5 pts) Extra-Credit : You may earn an extra 5 points by stating either The Fundamental Theorem of Integral Calculus or The Fundamental Theorem of Arithmetic.