1. (8 pts.) $f(x) = \dfrac{\sqrt{x^2-1}}{x-1}$ and $g(x) = x+1$.

(a) Find the domain of f.

$x \neq 1, \quad x^2-1 \geq 0$

$\text{Domain}(f) = (-\infty, -1] \cup (1, \infty)$

(b) Find the composition f(g(x)).

$$f(g(x)) = f(x+1) = \frac{\sqrt{(x+1)^2-1}}{(x+1)-1} = \frac{\sqrt{x^2+2x}}{x}$$

2. (12 pts.) Use the definiton of derivative to find the derivative of $f(x) = x^3$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \to 0} \frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{3x^2h+3xh^2+h^3}{h} = \lim_{h \to 0} 3x^2+3xh+h^2$$

$$f'(x) = 3x^2.$$

3. (6 pts.) A language student can memorize p(t) phrases in t hours of class. What are the units of $p'(t)$? Interpret $p'(4) = 6$.

Units of $p'(t)$: $\dfrac{\text{phrases}}{\text{hour of class}}$

$p'(4) = 6$ means that after 4 hours of class a language student can memorize 6 additional phrases for each additional hour of class.

4. (24 pts.) Find the derivatives of the following functions (you need not simplify):

(a) $f(x) = -\frac{8}{x} + \pi x^{-4} - 4x^3 + 100e$

$f'(x) = \frac{8}{x^2} - 4\pi x^{-5} - 12x^2.$

(b) $f(x) = (3x^5 - 18x)^4(5x^{-3} - 10x)$

$f'(x) = 4(3x^5 - 18x)^3(15x^4 - 18)(5x^{-3} - 10x) + (3x^5 - 18x)^4(-15x^{-4} - 10)$

(c) $f(x) = \frac{x^2 - 1}{20x + x^3}$

$f'(x) = \frac{(20x + x^3)(2x) - (x^2 - 1)(20 + 3x^2)}{(20x + x^3)^2}$

(d) $f(x) = \left(\frac{x^2 + 1}{1 - x^3}\right)^4$

$f'(x) = 4\left(\frac{x^2+1}{1-x^3}\right)^3 \left[\frac{(1-x^3)(2x) - (x^2+1)(-3x^2)}{(1-x^3)^2}\right]$

5. (12 pts.) When a certain ball is thrown into the air, its vertical distance above the ground at t seconds is given by $s(t) = -16t^2 + 128t = -16t(t-8)$ feet .

(a) Find the velocity of the ball.

$v(t) = s'(t) = -32t + 128 \ \frac{ft}{sec}.$

(b) Find the acceleration of the ball at t = 8.

$a(t) = s''(t) = v'(t) = -32 \frac{ft}{sec^2}$

$a(8) = -32 \ ft/sec^2.$

(c) Find the maximum height of the ball.

$v(t) = 0$ at $t = 4$ sec.

$s(4) = -16\,(4)(-4) = 256$ ft

(d) Find the impact velocity of the ball.

Impact is at 8 sec.

$v(8) = -32(8) + 128 = -128 \ \frac{ft}{sec}$

6. (10 pts.) If total costs are given by $C(x) = 10x + 12$ and total revenues are given by $R(x) = 18x - x^2$, both in thousands of dollars, where x is the number of units, find the break-even points.

$P(x) = R(x) - C(x) = 18x - x^2 - (10x+12) = 8x - x^2 - 12.$

$P(x) = -(x^2 - 8x + 12) = -(x-2)(x-6)$

$P(x) = 0$ for $x = 2$ and $x = 6$.

the break-even points at $x=2$ and $x=6$

You may earn up to 8 points extra credit by finding the maximum profit.

the maximum profit occurs at $x=4$

$P(4) = -2(-2) = 4$ thousand dollars.

7. (8 pts.) $f(x) = x + \sqrt{x}$.

(a) Find the instantaneous rate of change of f at x = 4.

$f'(x) = 1 + \frac{1}{2\sqrt{x}}$

$f'(\,) = 1 + \frac{1}{2 \cdot 2} = \frac{5}{4}$. This is the inst. r.o.c. of f at $x=4$.

(b) Find an equation of the tangent line at x = 1.

$f'(1) = 1 + \frac{1}{2} = \frac{3}{2}$. $P(1,2)$.

$y - 2 = \frac{3}{2}(x-1)$ or $y = \frac{3}{2}x + \frac{1}{2}$ is an equation of the tangent line at $x=1$

8. (12 pts.) A company manufactures cordless telephones and finds that its cost function (the total cost of manufacturing x telephones) is $C(X) = 400\sqrt{x} + 500$ dollars, where x is the number of telephones produced. Find the marginal cost, the average cost, the marginal average cost, and estimate the cost of producing the 65-th telephone.

$MC(x) = C'(x) = \frac{200}{\sqrt{x}}$

$AC(x) = \frac{C(x)}{x} = \frac{400\sqrt{x}+500}{x} = \frac{400}{\sqrt{x}} + \frac{500}{x}$

$MAC(x) = \left[\frac{C(x)}{x}\right]' = -\frac{1}{2}(400)x^{-\frac{3}{2}} - \frac{500}{x^2}$

The cost of the 65-th telephone is $C(65) - C(64)$.

$C(65) - C(64) \approx C'(64) = \frac{200}{\sqrt{64}} = \frac{200}{8} = 25$ dollars.

9. (8 pts.) Give examples of:

(a) A function f which is defined at x = 3, but discontinuous at x = 3.

$$f(x) = \begin{cases} x & \text{for } x \leq 3 \\ x+1 & \text{for } x > 3 \end{cases}$$

(b) A function g which is continuous at x = 1, but has a vertical tangent line at x = 1.

$g(x) = \sqrt{x-1}$