Math 0120 Sample Final Exam

Whitehead (November 28, 2005)

REMARK: This sample final exam contains 16 problems. The final exam that you will take on Wednesday December 14, 2005, from 2:00 p.m. to 3:50 p.m., will contain between 10 and 12 problems. Your instructor will announce the location of your final exam.

1. Fill in the blanks:

a. $\frac{d}{dx}\ln(f(x)) =$_____ b. $\frac{d}{dx}[f(x)\,g(x)] =$_____ c. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] =$_____

d. $\frac{d}{dx}x^n =$_____ e. $\frac{d}{dx}e^{ax} =$_____(a is a constant) f. $\int \frac{1}{x}\,dx =$_____

g. $\int e^{ax}dx =$_____$(a \neq 0)$ h. $\int x^n dx =$_____$(n \neq -1)$

2. Find the equation of the tangent line to the graph of the function

$$f(x) = x^3 + 2x^2 - 3x + 4 \quad \text{at} \quad x = -2.$$

3. Find the following limits SHOW YOUR WORK.

a. $\lim\limits_{x \to 1} \frac{x^2+2}{x+1}$ b. $\lim\limits_{x \to -2} \frac{x^2-4}{x+2}$

4. Let $f(x)$ be given by $f(x) = \begin{cases} 2-3x, & x \leq 1 \\ x+6, & x > 1 \end{cases}$ Is $f(x)$ continuous at $x = 1$? GIVE A REASON FOR YOUR ANSWER.

5. Differentiate each of the following functions. DO NOT SIMPLIFY.

a. $f(t) = \frac{t^3 - 3t + 4}{(2t^2 + 3t - 2)^2}$ b. $g(x) = (2\sqrt{x} + \ln x)\,(3x^2 + 2)$

c. $h(x) = (4x^3 + 2x^{3/2} - x^{-2} + 3x^{-4/3})^{-3}$ d. $k(t) = te^{5t} - \ln(3t^2 + 7) + e$

6. Given $f(x) = x^4 + 4x^3$, make sign diagrams for the first and second derivatives. Sketch the graph of $f(x)$ showing all relative extreme points and inflection points. Clearly indicate the intervals where the function is increasing, decreasing, concave up, and concave down.

7. It costs the USA Motor Company \$8000 to produce each automobile, and the fixed costs are \$20,000 per week. The company's price function is $p(x) = 22000 - 70x$, where $p(x)$ is the price at which exactly x automobiles will be sold in a week. How many automobiles should be produced each week to maximize profit? SHOW YOUR WORK.

8. A furniture store expects to sell 250 sofas per year. Each sofa costs the store \$400, and there is a fixed charge of \$500 per order. If it costs \$100 to store a sofa for a year, how large should each order be and how often should the orders be placed to minimize inventory costs? SHOW YOUR WORK.

9. For the given equation, use implicit differentiation to find $\frac{dy}{dx}$ <u>and</u> evaluate $\frac{dy}{dx}$ at the given values of x and y. SHOW YOUR WORK.

$$x^2 + y^2 = xy + 7; \quad x = 3, \ y = 2$$

10. A sum is invested at 6% interest compounded continuously. How long will it take for the investment to double in value?

11. Evaluate the following integrals:

a. $\int x^2 \ln(3x)\, dx$ b. $\int t\,(t^2 - 6)^7 dt$ c. $\int (z^{3/2} - 2z^{1/2} + z^{-1}) dz$ d. $\int \frac{x}{e^x}\, dx$

12. Find the area between the curves $y = x^2 - 1$ and $y = 2x - 1$.

13. A culture of bacteria is growing at the rate of $80e^{0.4t}$ cells per day, where t is the number of days since the culture was started. Suppose that the culture began with 100 cells. Find a formula for the total number of cells in the culture after t days.

14. Let $f(x,y) = \ln(xy) + e^{xy} + x^2 y$. Find the partial derivatives f_x, f_y, f_{xx}, and f_{yy}.

15. Find all relative extreme values of the function $f(x,y) = y^2 - x^3 + 12x - 4y$.

16. Minimize the function $f(x,y) = \sqrt{2x^2 + y^2 + 7}$ subject to the constraint $x - 2y = 9$.