We dedicate this paper to the memory of Professor G. J. F. R.

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Deviation and Analysis of Near Wall Models for Channel and Recirculating Flows
Introduction

Keywords: Large eddy simulation, near wall models, influence, boundary conditions for low velocities.

In this paper, we consider a problem in which the solution is not known exactly. In the second, free-slip conditions are recovered. Our goal here is not to develop new theories of problem solutions. Rather, we are interested in studying the behavior of the asymptotic limits of small Re with increasing Reynolds numbers and studying their asymptotic forms. We derive effective friction coefficients for on the wall.

\[ \mathbf{u} = \text{Re} \cdot \mathbf{n} \]

of the form:

| Work of the inertia and Maxwell, we develop such boundary conditions on the behavior of the unknown flow near the wall. Inspired by early
There is a good deal of computational experience with LES, which seems to predict Naver-Stokes equations well. Although not as applicable for the full Naver-Stokes equations, dependent boundary layers and the overall computational complexity must be reduced in order to make the LES useful in engineering situations. However, the computational cost of LES is higher than that of the Navier-Stokes equations for low Reynolds number and in the absence of the interaction between the flow and the walls, better boundary condition models have not been developed so far. Further, LES is to be used as a part of a prediction process. The latter type of problem is very common in engineering situations in which the interaction between the flow and the walls, better boundary condition models for the flow, are not available, and the use of LES is not very common in engineering applications. For problems of the type of problem in current practice, it is feasible to use LES. First, for problems with large scale eddies, and some other problems, the use of LES is more beneficial than the use of the Navier-Stokes equations. The latter type of problem is very common in engineering situations in which the interaction between the flow and the walls, better boundary condition models for the flow, are not available, and the use of LES is not very common in engineering applications. For problems of the type of problem in current practice, it is feasible to use LES. First, for problems with large scale eddies, and some other problems, the use of LES is more beneficial than the use of the Navier-Stokes equations.
The correct NWV must ultimately depend on the other selected, the approach.

Several other choices are also commonly used, see e.g. [61]. Although

\[ f \star g = f \quad d \star g = d \quad n \star g = n \]

by convolution.

Extended all functions outside \( \mathbb{R} \) by zero, the index edles are then defined to be the Gaussian kernel, where \( 0 < \varepsilon < \infty \) is typically chosen to be 6. Have

\[
(2) \quad \left( \frac{1}{\varepsilon} \right)^{d/2} \exp \left( \frac{-x^2}{2\varepsilon^2} \right) = (x)^{d/2}
\]

Let

\[
p \geq d, \quad \exists \text{ such that } \frac{1}{p} \leq c
\]

The velocity decomposition tensor is given by

\[
\theta \times [L, 0] \quad \text{in } 0 = \mathbf{n} \cdot \Delta
\]

\[
\theta \times [L, 0] \quad \text{on } 0 = \mathbf{n}
\]

\[
\theta \times [L, 0] \quad \text{in } \mathbf{J} = d\Delta + \mathbf{n} (\Delta \cdot \mathbf{n}) + (\mathbf{n} \cdot \Delta_0 - 2\mathbf{n})
\]

Incompressible Navier-Stokes equations.
The boundary conditions (3) can be interpreted easily into a finite el-

dition of the normal stress at the wall will result in the following sum of normal stresses and boundary condition which must satisfy: \( \mathbf{u}(\mathbf{n}) \cdot \mathbf{n} + \mathbf{\sigma}_n \cdot \mathbf{n} \) and \( 0 = \mathbf{n} \cdot \mathbf{n} \) on \( \partial \Omega \), where \( \mathbf{n} \) is the outward unit normal.

The most commonly used boundary condition is \( \mathbf{n} = 0 \) on \( \partial \Omega \). It is easy to see that this is not consistent, see Figure 1. If \( \mathbf{n} \) also does not agree with our physical intuition of large eddies, we will also use a consistent term, \( \mathbf{n} \cdot \mathbf{n} \). We will construct terms \( \mathbf{\sigma}_n \cdot \mathbf{n} \) for the consistent term. \( \mathbf{\sigma}_n \) and \( \mathbf{n} \) are built into the energy equation of large eddies, as they are included as examples and can.

**Figure 1:** everywhere the velocity at the boundary does not give homogeneous

results. To solve any such system, an \( \mathbf{n} \) must be specified.
Section 2 considers the important question of near-well modelling for

\[
(y, \Phi) \frac{\partial}{\partial y} \sqrt{\gamma} \sim (y, \Phi) \frac{\partial}{\partial y} \sqrt{\gamma}
\]

where \( \Phi \) is a function which is uniformly positive and bounded in both \( y \) and

and show

In the laminate case, we give the optimal value of \( \gamma \) and can help to optimize parameter selection in the penalty methods

Since laminate flows can also be under-resolved, this case is not without

introduce the ideas and methods, we consider the 2D laminate case in Section

A 2D laminate flow will follow. After briefly considering some needed formulas in Section 6, we

as noted above, the behavior of \( \Gamma \) depends on the behavior of \( n \),

\[
\frac{\partial (\gamma)}{\partial y} = \frac{T}{T-1} \frac{\partial y}{\partial y} \sim \gamma
\]

The friction coefficient \( \gamma \) is defined as a ratio of the shear stress to the shear force.

\[
\text{microscopic length scale} \sim \gamma
\]

\[
\frac{\text{macroscopic length scale}}{\text{microscopic length scale}} \sim \gamma
\]

Maxwell’s derivation, the friction coefficient \( \gamma \) is defined by

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\[
\gamma \sim \frac{\text{macroscopic length scale}}{\text{microscopic length scale}}
\]

\[
\text{Maxwell} \sim \frac{\text{macroscopic length scale}}{\text{microscopic length scale}}
\]

The condition \( \gamma \) is a mathematical expression of the realignment of the

dimensionless channel flows across a step and is presented which study the ini-

\[
\text{Maxwell} \sim \frac{\text{macroscopic length scale}}{\text{microscopic length scale}}
\]
A laminar boundary layer with uniform suc-

tion in 2-D
\[
\cdot \left\{ \left[ \left( \frac{\partial \varphi}{\partial y} - \frac{\partial \varphi}{\partial x} \right)^2 \right] \cdot \left( \frac{\partial \varphi}{\partial y} \frac{\partial \varphi}{\partial x} \right) \right\} \cdot \frac{\partial}{\partial x} = - \frac{\partial}{\partial x} \left[ \left( \frac{\partial \varphi}{\partial y} \frac{\partial \varphi}{\partial x} \right) \right] \cdot \frac{\partial}{\partial x}
\]

This integral can be evaluated using (52), and we obtain (53)

\[
\cdot \left[ \frac{\varphi _p}{\varphi _q} \left( \left( \frac{\partial \varphi}{\partial y} \frac{\partial \varphi}{\partial x} \right) \right) \cdot \frac{\partial}{\partial x} \left[ \left( \frac{\partial \varphi}{\partial y} \frac{\partial \varphi}{\partial x} \right) \right] \cdot \frac{\partial}{\partial x}
\]

\[
\cdot \frac{\partial}{\partial x} \left[ \left( \frac{\partial \varphi}{\partial y} \frac{\partial \varphi}{\partial x} \right) \right] \cdot \frac{\partial}{\partial x}
\]

We start by computing (0, x) \bullet \eta = \frac{\partial \varphi}{\partial y} \frac{\partial \varphi}{\partial x}

\[
\cdot \frac{\partial}{\partial x} \left[ \left( \frac{\partial \varphi}{\partial y} \frac{\partial \varphi}{\partial x} \right) \right] \cdot \frac{\partial}{\partial x}
\]

\[
\cdot \frac{\partial}{\partial x} \left[ \left( \frac{\partial \varphi}{\partial y} \frac{\partial \varphi}{\partial x} \right) \right] \cdot \frac{\partial}{\partial x}
\]

We conclude that \[(0, \varphi) \cdot \eta = \frac{\partial \varphi}{\partial y} \frac{\partial \varphi}{\partial x}
\]

\[
\cdot \frac{\partial}{\partial x} \left[ \left( \frac{\partial \varphi}{\partial y} \frac{\partial \varphi}{\partial x} \right) \right] \cdot \frac{\partial}{\partial x}
\]

\[
\cdot \frac{\partial}{\partial x} \left[ \left( \frac{\partial \varphi}{\partial y} \frac{\partial \varphi}{\partial x} \right) \right] \cdot \frac{\partial}{\partial x}
\]

\[
\cdot \frac{\partial}{\partial x} \left[ \left( \frac{\partial \varphi}{\partial y} \frac{\partial \varphi}{\partial x} \right) \right] \cdot \frac{\partial}{\partial x}
\]

We conclude the model situation that \([0, \varphi) \cdot \eta = \frac{\partial \varphi}{\partial y} \frac{\partial \varphi}{\partial x}
\]

\[
\cdot \frac{\partial}{\partial x} \left[ \left( \frac{\partial \varphi}{\partial y} \frac{\partial \varphi}{\partial x} \right) \right] \cdot \frac{\partial}{\partial x}
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We conclude the model situation that \([0, \varphi) \cdot \eta = \frac{\partial \varphi}{\partial y} \frac{\partial \varphi}{\partial x}
\]

\[
\cdot \frac{\partial}{\partial x} \left[ \left( \frac{\partial \varphi}{\partial y} \frac{\partial \varphi}{\partial x} \right) \right] \cdot \frac{\partial}{\partial x}
\]
\[
\frac{y}{z} = \frac{y}{z} e^{y/\sqrt{z}}
\]

Consequently,
\[
\frac{y}{z} = \frac{y}{z} e^{y/\sqrt{z}}
\]

and for fixed values, we have
\[
\text{Proposition 2.1: Let } \Re(e) \text{ be given as in (10). For fixed values, we have:}
\]
\[
(\frac{y}{z}) / \Re(e) \mid_{0}^{\infty}
\]

Note that, for \( y \in \mathbb{R} \), we have
\[
(0, x) = (0, x) e^{y/\sqrt{z}}
\]

\[
(\Re(e) / \Re(e) \mid_{0}^{\infty}
\]

The friction coefficient for the laminar boundary layer wall is given by
\[
(\Re(e) / \Re(e) = \frac{1/2}{(0, x) e^{y/\sqrt{z}}}
\]

which gives
\[
\left\{ \left( \frac{\Re(e) / \Re(e) - \Re(e) / \Re(e) \mid_{0}^{\infty}}{\Re(e) / \Re(e) \mid_{0}^{\infty}} \right) \right\}
\]

\[
\left\{ \left( \frac{\Re(e) / \Re(e) - \Re(e) / \Re(e) \mid_{0}^{\infty}}{\Re(e) / \Re(e) \mid_{0}^{\infty}} \right) \right\}
\]

\[
\left\{ \left( \frac{\Re(e) / \Re(e) - \Re(e) / \Re(e) \mid_{0}^{\infty}}{\Re(e) / \Re(e) \mid_{0}^{\infty}} \right) \right\}
\]

\[
\left\{ \left( \frac{\Re(e) / \Re(e) - \Re(e) / \Re(e) \mid_{0}^{\infty}}{\Re(e) / \Re(e) \mid_{0}^{\infty}} \right) \right\}
\]

\[
\left\{ \left( \frac{\Re(e) / \Re(e) - \Re(e) / \Re(e) \mid_{0}^{\infty}}{\Re(e) / \Re(e) \mid_{0}^{\infty}} \right) \right\}
\]

\[
\left\{ \left( \frac{\Re(e) / \Re(e) - \Re(e) / \Re(e) \mid_{0}^{\infty}}{\Re(e) / \Re(e) \mid_{0}^{\infty}} \right) \right\}
\]

The numerator in (9) can be computed directly from (2) using (30). We obtain
\[
(9) = \left\{ \left( \frac{\Re(e) / \Re(e) - \Re(e) / \Re(e) \mid_{0}^{\infty}}{\Re(e) / \Re(e) \mid_{0}^{\infty}} \right) \right\}
\]

In particular, we have
The asymptotic behaviors are illustrated in Figure 2.

The friction coefficient for fixed Re, asymptotic to Re for constant $\theta$, with $v = 6, V_0 = 1$, $r = 1$, at high $Re$, is $f(\theta, \gamma)$, which is illustrated in Figure 2. Laminar boundary layer: $Re$ of $\theta$. Laminar boundary layer: $Re$ of $\theta$.
\[ \tilde{y} = y \sim \mathcal{L} \]

By \( \tilde{y} = y \sim \mathcal{L} \), the free stream velocity of the most common laws is given

\[ 0 \leq x < \frac{\mu}{\rho} \left( \frac{\rho x^2}{\nu} \right) x^3 \leq 0 \Rightarrow (x)u = (x)u \]

where the boundary layer thickness is given by

\[ h > u \quad \text{for } 0 = n = \frac{\mu}{\rho} \cdot \left( \frac{\rho x^2}{\nu} \right) \frac{\rho x^2}{\nu} \]

\[ \begin{cases} h > u \\ l \leq h \leq 0 \end{cases} \]

\[ 0 = h \cdot 0 \leq x \end{cases} \]

Consider the full plane of the velocity is given by

\[ \{(x, y) : (x, y) \in \mathbb{R}^2, \forall x \in \Omega \} \]

provided the ensemble average of the velocity is given by

\[ \langle (x, y) \rangle \sim \mathcal{L} \]

where \( h \) is the local mesh width near \( \Omega \).

\[ \langle (x, y) \rangle \sim \mathcal{L} \]

penny parameter \( e \) in the methods of \( e, g, c, \) should be

\[ \langle (x, y) \rangle \sim \mathcal{L} \]

Further, they give important insight into parameter selection in penalty method.

The formula \( (11) \) and \( (11) \) agree with the formula \( \Omega \) of Maxwell. The formula \( T \cdot H \sim 0.3 \) of Maxwell agrees with the formula \( \Omega \) of Maxwell.

Remark 2.1: The friction laws are derived herein and in the following section are linear and thus not appropriate for recirculating flows. Nevertheless, they serve necessary steps to deriving the non-linear friction laws for
\begin{equation}
\tag{16}
\frac{(z', 0', x')}{(z, 0, x)} \frac{\nu}{\nu_0} \cdot \partial \mathbf{H} = (\nu \mathbf{H}, 0) \mathbf{E}
\end{equation}

\begin{equation}
\{0 = \rho\} \quad 0 = \frac{\rho}{\rho_0} \cdot \partial \mathbf{H} = \nu(\nu \mathbf{H}, 0) \mathbf{E}
\end{equation}

Thus, this simplifies to

\begin{equation}
\nu > 0 \quad \sigma = \frac{\sigma}{\sigma_0} \quad \frac{\nu}{\nu_0} \quad \mathbf{H} \quad \mathbf{E}
\end{equation}

\begin{equation}
\begin{cases}
\mathbf{H} & \mathbf{E} \quad \mathbf{H} \quad \mathbf{E}
\end{cases}
\end{equation}

Now, we define a \textit{x-averaged} vector by

\begin{equation}
\langle \mathbf{H} \rangle = \frac{1}{t} \int_0^t \mathbf{H}(\cdot, \mathbf{x}) \, dt
\end{equation}

Boundary layer thickness by

\begin{equation}
\langle \mathbf{H} \rangle = \langle \mathbf{H} \rangle = \frac{1}{t} \int_0^t \mathbf{H}(\cdot, \mathbf{x}) \, dt
\end{equation}

In order to handle this situation, we have to

\begin{equation}
\{0 < \rho, \rho \in (z', \rho', x)\} = \mathbb{R}
\end{equation}

We consider the modulated situation of a reference plane of non-dimensional
The ship coefficient is now be computed by

\[
\begin{aligned}
(18) \quad & \left[ \left( \frac{g}{L} \right) \cdot \left( \frac{v}{\nu} \right) \right] \cdot \left[ \int_{0}^{1} \frac{u}{v} \right] \cdot \left( \frac{\nu}{\nu} \right) \cdot \left( \frac{v}{v} \right) \cdot \frac{v}{v} \\
& = \left[ \int_{0}^{1} \frac{u}{v} \right] \cdot \left( \frac{\nu}{\nu} \right) \cdot \left( \frac{v}{v} \right) \cdot \frac{v}{v}
\end{aligned}
\]

\[
\int_{0}^{1} \int_{0}^{1} \frac{u}{v} \quad \left( \int_{0}^{1} \frac{u}{v} \right) \quad \left( \frac{\nu}{\nu} \right) \quad \left( \frac{v}{v} \right) \quad \frac{v}{v}
\]

A straightforward computation using (23) of (23) yields

\[
\frac{h_{0}}{n_{0}} * \eta_{0} = \frac{h_{0}}{n_{0}}
\]

If so as to remain one week derivative, i.e.,

To compute the integral in (16), we note first that differentiation and

\[
\begin{aligned}
(11) \quad & \left[ \left( \frac{g}{L} \right) \cdot \left( \frac{v}{\nu} \right) \right] \cdot \left( \frac{v}{v} \right) \cdot \frac{v}{v} \\
& = \left[ \int_{0}^{1} \frac{u}{v} \right] \cdot \left( \frac{\nu}{\nu} \right) \cdot \left( \frac{v}{v} \right) \cdot \frac{v}{v}
\end{aligned}
\]

\[
\int_{0}^{1} \int_{0}^{1} \frac{u}{v} \quad \left( \int_{0}^{1} \frac{u}{v} \right) \quad \left( \frac{\nu}{\nu} \right) \quad \left( \frac{v}{v} \right) \quad \frac{v}{v}
\]

We obtain, using (23) of (23)
\[ 0 < a, \quad 0 = \left[ \left( \frac{X}{v} \right)_{\text{pdf}} - 1 \right] \frac{X - x}{\ln v} \]

of Bernoulli-Hospitals proves the theorem that the rule of the denominator tends to 1. Applying these three times the rule \((v \ln v) / (1 + v)\) to 1.

Let \( a \) be fixed and consider the last factor in (19). The application of

\[ (22) \quad \phi \] \[ (22) \quad \psi \]

From (24) and by the definition of the Gamma function follows

\[ (12) \quad \phi \] \[ (12) \quad \psi \]

Identity if \( g \) is constant, then

\[ (20) \quad \phi \] \[ (20) \quad \psi \]

Identity if \( H \) is constant, then

**Proposition 3.1.** Let \((v, H)\) be given as in (19). If \( H = \) is constant, then

**Remark 3.1.** Considering the 0-th power law in \( z \) under the same geo-

\[ \left[ \left( \frac{a}{\ln v} \right)_{\text{pdf}} - 1 \right] \frac{a_{\text{pdf}} - 1}{\ln v} + \left[ \left( \frac{a_{\text{pdf}}}{1 + v} \right) \frac{1}{v \ln v} \right] - \left( \frac{1}{1 + v} \right) \]

\[ \left[ \left( \frac{a_{\text{pdf}}}{1 + v} \right) \frac{1}{v \ln v} \right] - \left( \frac{1}{1 + v} \right) \]

\[ \frac{a_{\text{pdf}}}{v \ln v} \]

and

\[ \phi \] \[ \psi \]
Remark 3.2. It is interesting that the limiting forms of the optimal linear

\[
\begin{align*}
\text{Re} & = \infty, \\
\text{Re} & = 0.
\end{align*}
\]

Figure 3: 1/7-th power law boundary layer: left: behavior of \( \delta \), right with respect to \( \Re \) constant; right: behavior of \( \Re \), right with respect to \( \delta \) constant. \( \delta = 0 \).}

The asymptotic behaviors in the case \( \alpha = 0 \) are illustrated in Figure 3.

The derivatives of the functions \( f(x) \) with respect to the upper boundary, the outer edge, and the inner edge are equal to zero. Thus, the second term in the denominator tends to zero and hence the last term in the numerator becomes negligible. Therefore, the solution becomes

\[
\frac{1}{\Re} \left( \frac{\text{const}}{\Re} \right).}
\]
A Near Wall Model for Recirculating Flows
\[ \mu \left[ \left( \frac{\nu - \nu_0}{\nu_0 \cdot \mathbf{n}} \right) \exp \left( - \left\lceil \frac{\nu - \nu_0}{\frac{\nu - \nu_0}{\nu_0} \cdot \mathbf{n}} \right\rceil - 1 \right) + \left( \frac{\nu - \nu_0}{\nu_0 \cdot \mathbf{n}} \right) \right] \sum_{\nu=1}^{\infty} \sum_{N=1}^{\infty} \frac{\nu_0}{N} \left( \frac{\nu}{\nu_0} \right) \frac{1}{\nu_0} \right) \frac{1}{\nu_0} \frac{1}{\nu_0} \]
\[ z = \lim_{n \to \infty} \left( \sqrt[n]{a} \right) \]

Table 1: The function \( z(t) \) and its exponential approximation according to

![Graph showing exponential approximation]

<table>
<thead>
<tr>
<th>( q )</th>
<th>( a )</th>
<th>( z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.170289</td>
<td>0.1</td>
</tr>
<tr>
<td>0.01</td>
<td>0.083473</td>
<td>0.1</td>
</tr>
<tr>
<td>0.10</td>
<td>0.342369</td>
<td>0.0</td>
</tr>
<tr>
<td>0.20</td>
<td>0.268180</td>
<td>0.1</td>
</tr>
<tr>
<td>0.40</td>
<td>0.497275</td>
<td>0.0</td>
</tr>
<tr>
<td>0.60</td>
<td>0.961519</td>
<td>0.1</td>
</tr>
<tr>
<td>1.00</td>
<td>1.428564</td>
<td>0.0</td>
</tr>
</tbody>
</table>

The approximation is presented in Figure 4. Computed with \( N = 1000 \) using Newton's method. An illustration of the optimal parameters for some intervals in Table 1. These parameters were be solved iteratively, e.g., by Newton's method. We give some examples for the necessary condition for a minimum, that the derivatives with respect to \( a \) and \( q \) vanish. Leads to a non-hyper system of two equations.
In Table 2, the number of degrees of freedom for various levels are given.

<table>
<thead>
<tr>
<th>Levels</th>
<th>Pressure</th>
<th>Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1100</td>
<td>236</td>
</tr>
<tr>
<td>2</td>
<td>4248</td>
<td>722</td>
</tr>
<tr>
<td>3</td>
<td>1665</td>
<td>906</td>
</tr>
<tr>
<td>4</td>
<td>1928</td>
<td>4765</td>
</tr>
<tr>
<td>5</td>
<td>2074</td>
<td>4906</td>
</tr>
<tr>
<td>6</td>
<td>4768</td>
<td>4906</td>
</tr>
</tbody>
</table>

The domain of the two-dimensional flow across a step is presented in Figure 3. Here, we present results for a parabolic inflow profile, which is deformed by a step. The flow is presented in Figure 2. The number of degrees of freedom for various levels are given in Table 2. We study the dependence of the position of the reattachment condition. We present the results for the simulation model with no-step condition. We present the results for the simulation model with no-step condition. We present the results for the simulation model with no-step condition. We present the results for the simulation model with no-step condition. We present the results for the simulation model with no-step condition. We present the results for the simulation model with no-step condition. We present the results for the simulation model with no-step condition.

5 Numerical Test
The reallocation point is defined by the change of the sign of the ran-

Figure 6: Two dimensional channel with a step on coarsest mesh (level 1).

Figure 7: Domain of two dimensional channel with a step.
For instance, the Smagorinsky model constant $c_s$ is a priori input, and although the Smagorinsky model is widely used, it has some drawbacks.

When our calculations, we fixed the Smagorinsky constant $c_s$ to be $0.01$. During our calculations, we fixed the deformation tensor and $c_s \in [1, 10]$, e.g., see Segall (26)

$$d \sqrt{(n)_{||}} \beta_{SC} = \mu$$

![Figure 6: Two dimensional channel with a step finer mesh (level 3)](image6)

![Figure 8: Two dimensional channel with a step coarse mesh (level 2)](image8)
data with no clean comparison.

NW Y's leads to a valid conclusion while varying both would lead to a lot of
11 and 12. However, keeping the coarse model fixed and testing various
into the flow, we shall see some numerical tests on this as seen in the figures.

However, another drawback of this model is that it introduces too much diffusion
this single constant is not capable of representing correctly various temporal

Figure 9: Two dimensional channel with a step, mesh grid (level 4).
Reattachment Points and Time, $\text{Re}=600$, $\beta=1.0$, $\delta=6.7$ (Level1), $\delta=0.83$ (Level4), $c_s=0.01$
\begin{equation}
\left( e^{(q + x\nu)} - 1 \right) \frac{\nu}{p} \exp \left( e^{(q + x\nu)} - 1 \right) = (q + x\nu) \frac{xp}{p}
\end{equation}

The derivative of the error function is easily calculated from (4) is to be
\begin{equation}
\left( (\nu + q)^{\nu + 1} \right) \frac{\nu}{p} \exp \left( e^{(q + x\nu)} - 1 \right) \int_{0}^{\infty} e^{x \nu} \exp \left( e^{(q + x\nu)} - 1 \right) \, dx
\end{equation}

and
\begin{equation}
\frac{\nu}{p} \exp \left( e^{(q + x\nu)} - 1 \right) \int_{0}^{\infty} e^{x \nu} \exp \left( e^{(q + x\nu)} - 1 \right) \, dx
\end{equation}

where \( \exp \) denotes the error function. We shall call that
\begin{equation}
0 < \nu \quad \left( \nu + q \right)^{\nu + 1} \frac{\nu}{p} \exp \left( e^{(q + x\nu)} - 1 \right) \int_{0}^{\infty} e^{x \nu} \exp \left( e^{(q + x\nu)} - 1 \right) \, dx
\end{equation}

I holds

The appendix provides collection of some formulas which have been used.

\textbf{Appendix 6}

\begin{itemize}
\item Simulation Plus Modeling Simulation
\item Further study and test of this approach are thus well-understood.
\end{itemize}

Clearly, the Smagorinsky model is too simplistic. This should address, the underresolved NSP + SWF simulation problems. Indeed, the Smagorinsky model is not very well understood. However, it is also clear that the SWP simulates the main point of the model. The NSP + SWF + eddy model remains separate and evolve remain attached and align steadily. However, the Smagorinsky model is too simplistic. This should address, the underresolved NSP + SWF simulation problems.

The results depicted in Figure 10 are very informative and give strong
Figure 11: Two dimensional channel with a step, the streamlines of the solution for No-slip condition on finest mesh (left) and Smagorinsky-No-slip on coarsest mesh (right) at time $t=40$, $Re=600$. 

NSE+NOSLIP, $Re=600$, At $t=40$, $\beta = 1.0$, $\delta = 0.8$, Level=4

NSE+SMA+NOSLIP, $Re=600$, At $t=40$, $\beta = 1.0$, $\delta = 6.7$, Level=1
Figure 12: Two dimensional channel with a step, the streamlines of the solution for slip with friction condition on coarsest mesh (left) and Nagorinsky - slip with friction on coarsest mesh (right) at time $t = 40$, $Re = 600$. 

NSE+SWF, $Re=600$, At $t=40$, $\beta = 1.0$, $\delta = 6.7$, Level=1 

NSE+SMA+SWF, $Re=600$, At $t=40$, $\beta = 1.0$, $\delta = 6.7$, Level=1
References

(32) \[ \left[ \left( \frac{e^{qy}}{1+\varphi} \right) \right] \left[ \left( \frac{e^{qy}}{1+\varphi} \right) \right] = \int_0^\infty x p x (e^{qy} - e^{qy}) \, dx \]

Combining (32) and (33) gives

(33) \[ x p^t e^{-\varphi/1+\varphi} = \int_0^\infty x p x (e^{qy} - e^{qy}) \, dx \]

The substitution \( t = q/\varphi \) yields

(33) \[ x p^t e^{-\varphi/1+\varphi} = \int_0^\infty x p x (e^{qy} - e^{qy}) \, dx \]

The third type of integral which will be needed here is

(33) \[ \left[ \left( \frac{e^{qy}}{1+\varphi} \right) \right] \left[ \left( \frac{e^{qy}}{1+\varphi} \right) \right] = \int_0^\infty \left( e^{qy} - e^{qy} \right) \, dx \]

with \( q < 0 \), see Abramowitz and Stegun [1], Chapter 7.4. It follows that

(33) \[ \left[ \left( \frac{e^{qy}}{1+\varphi} \right) \right] \left[ \left( \frac{e^{qy}}{1+\varphi} \right) \right] = \int_0^\infty \left( e^{qy} - e^{qy} \right) \, dx \]

A second useful type of integral is
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