**Problem 1:** Let $V$ be the set of all (ordered) pairs of real numbers. Let $x = (a_1, a_2)$ and $y = (b_1, b_2)$ be elements of $V$ and the addition and multiplication be defined such that

\[
x + y = (a_1 + b_1, a_2 + b_2)
\]

\[
kx = (ka_1, 0)
\]

\[
0 = (0, 0)
\]

\[
-x = (-a_1, -a_2)
\]

Is $V$ with the defined operations a linear space? Why?

**Problem 2:** Consider the vector space $P$ of all polynomials over $\mathbb{R}$ and the subset $V$ of $P$ consisting of those polynomials for which

(a) $x(t) \geq 0$ whenever $0 \leq t \leq 1$

(b) $x(t) = x(1-t)$ for all $t$

In which of these cases is $V$ a linear space? Justify.

**Problem 3:** Let $Y$ and $Z$ be two subspaces of a linear space $X$. Show that $Y + Z$ and $Z \cap Y$ are subspaces of $X$.

**Problem 4:** Under what conditions on the scalar $k$ are the vectors $(k,1,0)$, $(1,k,1)$, $(0,1,k)$ in $\mathbb{R}^3$ linearly dependent? What is the answer for $\mathbb{Q}^3$ (in place of $\mathbb{R}^3$)?

**Problem 5:** Let $X$, $Y$ and $Z$ be subspaces of a linear space. Show that the subspaces $(X \cap Y) + (X \cap Z)$ and $X \cap (Y + Z)$ are not necessarily identical.

**Problem 6:** Show that if $x_1, x_2, \ldots, x_n$ are linearly independent then $x_i \neq 0$ for $i = 1, 2, \ldots, n$.

**Problem 7:** Let $Y$ and $Z$ be two subspaces of a linear space $X$. Show that

\[
\dim Y + \dim Z = \dim(Y + Z) + \dim(Y \cap Z)
\]