Problem 1:
Let $P$ be a projector. Show that the complementary projector $I - P$ projects onto $\text{null}(P)$.

Problem 2:
Let $B$ be a $3 \times 3$ matrix. Each of the following operations on $B$,
1. doubling of column 1
2. halving row 3
3. adding row 3 to row 1
4. interchanging columns 2 and 1
can be expressed as a multiplication of $B$ by a matrix $E_i$. Find such matrices $E_i$. Write the result of application of all four operations on $B$ as a product of three matrices $ABC$.

Problem 3:
Show that if $Q$ is orthogonal and upper triangular then it is diagonal, with each element on the diagonal being either 1 or $-1$.

Problem 4:
Consider the matrix $A = I + uv^T$. Show that if $A$ is invertible, its inverse is given by $A^{-1} = I + c|v|v^T$. Find the value of $c$. What condition do the vectors $u$ and $v$ satisfy when $A$ is singular?

Problem 5:
Let $P$ be the orthogonal projector onto the span of $p$ and $q$, where $p$ and $q$ are orthonormal vectors. Show that $I - P$ is the product of two projectors, $I - pp^T$ and $I - qq^T$.

Problem 6:
Find an orthogonal projector $P$ that projects onto $\text{null}(A)$ where

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 0 & 1 & 1 \\ -1 & 2 & 0 \end{bmatrix}$$

Problem 7:
Calculate the Householder reflector that takes the vector $[1 \ 0 \ 2 \ -1]^T$ into $[\sqrt{6} \ 0 \ 0 \ 0]^T$.

Problem 8:
Find the distance of the point $b = [1 \ 1 \ 0 \ 2]^T$ from the hyperplane defined as the span of $[1 \ 0 \ 2 \ 4]^T$ and $[0 \ -2 \ 1 \ 1]^T$, using least squares minimization.
Problem 9:
Compute the QR factorization of the following matrix
\[
A = \begin{bmatrix}
-1 & 1 & 0 \\
1 & -2 & 1 \\
0 & 1 & -1
\end{bmatrix}
\]

Problem 10:
Using QR factorization find the solution of the system of equations
\[
\begin{bmatrix}
1 & 2 & 3 \\
2 & 3 & 4 \\
3 & 4 & 6
\end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}
\]