Problem 1:
Each of the following statements is either true or false. Give a proof for those that are true and give a counterexample for those that are false:

a) If $\lambda$ is an eigenvalue of $A$ then $\lambda - \mu$ is an eigenvalue of $A - \mu I$.

b) If $A$ is real and $\lambda$ is an eigenvalue of $A$ then $-\lambda$ is also an eigenvalue of $A$.

c) If all the eigenvalues of $A$ are zero, then $A = 0$.

Problem 2:
Find the eigenvalue decomposition $A = X \Lambda X^{-1}$ of the following matrix:

$$A = \begin{bmatrix}
-1 & 0 & -4 & 0 \\
0 & 2 & -1 & 0 \\
0 & 0 & 3 & 0 \\
0 & 3 & 3 & -1
\end{bmatrix}$$

Problem 3:
Find the orthogonal diagonalization $A = Q \Lambda Q^T$ for the following symmetric matrix $A$:

$$A = \begin{bmatrix}
4 & -2 & 1 \\
-2 & 4 & 1 \\
1 & 1 & 1
\end{bmatrix}$$