**Math 1080: Spring 2005**  
**Homework #10  (due April 15)**

**Problem 1:**
Let $Q$ and $R$ be the QR factors of a symmetric tridiagonal matrix $H$. Show that the product $K = RQ$ is again a symmetric tridiagonal matrix.

(Hint: Prove the symmetry of $K$. Show that $Q$ has Hessenberg form and that the product of an upper triangular matrix and a Hessenberg matrix is again a Hessenberg matrix. Then use the symmetry of $K$.)

**Problem 2:**
Determine one eigenvalue of the following matrix using Rayleigh Quotient iteration, starting with initial guess $v^{(0)} = [0 \ 1]^T$. Terminate iteration after 3 steps, i.e., after you obtain $\lambda^{(3)}$. What is the approximate eigenvector $v^{(3)}$? What is the error of $\lambda^{(3)}$?

\[
A = \begin{bmatrix}
3 & -2 \\
-2 & 6
\end{bmatrix}
\]

**Problem 3:**
Perform the first two steps of the QR algorithm (i.e., compute $A^{(2)}$ and $\tilde{Q}^{(2)}$) for the following matrix. How close are the diagonal elements of $A^{(2)}$ to the eigenvalues of $A$?

\[
A = \begin{bmatrix}
2 & -1 & 0 \\
-1 & 2 & -1 \\
0 & -1 & 2
\end{bmatrix}
\]

**Computer Assignment 6:**

a) Write a MATLAB function $[Q, L] = qralg(A)$ that computes the eigenvalues and eigenvectors of a square, symmetric $m \times m$ matrix $A$ using QR algorithm. The output variables are the $m \times m$ orthogonal matrix $Q$ which columns are the eigenvectors of $A$ and $m \times m$ matrix $L$ that has the corresponding eigenvalues of $A$ on the main diagonal. The program should terminate iteration when the norm of the offdiagonal elements of $A^{(k)}$, i.e., $\text{norm}(A - \text{diag}(\text{diag}(A)))$, is smaller than $10^{-6}$.

b) Use the function $qralg$ to calculate the eigenvalues and eigenvectors of

\[
A = \begin{bmatrix}
2 & 6 & 4 & -4 & -5 & -10 \\
6 & 12 & -2 & 9 & 5 & 9 \\
4 & -2 & 0 & -1 & -3 & 14 \\
-4 & 9 & -1 & 14 & -6 & 8 \\
-5 & 5 & -3 & -6 & 2 & 8 \\
-10 & 9 & 14 & 8 & 8 & 8
\end{bmatrix}
\]

Record the number of iterations needed to achieve the desired accuracy.